Hybridization of quantum and thermal effect in Ising machines

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Introduction

- Some combinatorial optimization problems can be represented by Ising model or Quadratic Unconstrained Binary Optimization (QUBO).
  A. Lucas, Frontiers in Physics, **2**, 5 (2014).

$$E = -\sum_{i \in V} h_i s_i - \sum_{(i,j) \in E} J_{ij} s_i s_j , \qquad s_i \in \{+1, -1\}. \quad \text{(Ising model)}$$

$$E = \sum_{1 \le i \le j \le N} Q_{ij} x_i x_j , \qquad x_i \in \{0,1\}. \quad \text{(QUBO)}$$

- Ising machines which can solve combinatorial optimization problems efficiently have been developed.

- Toward the development of more high-performance Ising machines, internal algorithms should be sophisticated.

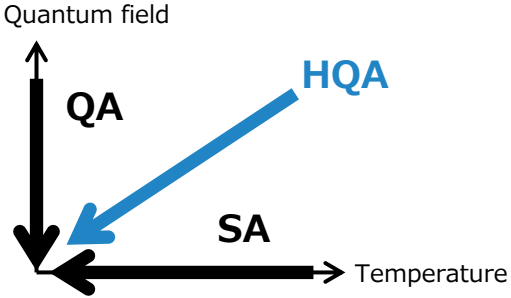
- In recent preceding studies, the performance of quantum annealing enhances by the hybridization of temperature and quantum effect.
  N.G. Dickson et al., **4**, 1903 (2013). T. Shirai, T. Kadowaki, and S. Tanaka, Japan-Netherlands Quantum Conference, 2019.

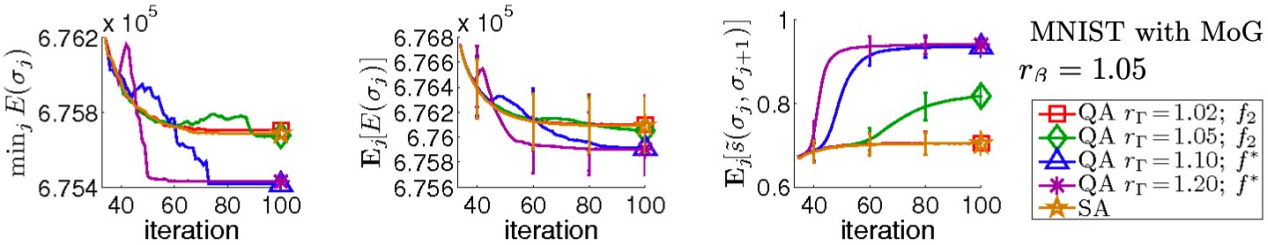
Background

- Kurihara, Tanaka, and Miyashita proposed hybrid annealing in which temperature and quantum field decrese simultaneously.
  K. Kurihara, S. Tanaka, and S. Miyashita, UAI2009
  I. Sato, K. Kurihara, S. Tanaka, H. Nakagawa, and S. Miyashita, UAI2009
  I. Sato, S. Tanaka, K. Kurihara, S. Miyashita, and H. Nakagawa, Neurocomputing 121, 523 (2013).

- They demonstrated the above algorithm to some machine learning problems such as clustering, variational Bayes inference, Chinese restaurant process.

- They found that the hybrid quantum annealing (HQA) with schedule function in which first the temperature effect and then the quantum effect are gradually weakened seems to be more advantageous. HQA is regarded as the hybrid algorithm of simulated annealing (SA) and quantum annealing (QA).





K. Kurihara, S. Tanaka, and S. Miyashita, UAI2009

Proposed method

1

We consider Ising Model which represents combinatorial optimization problems.

$$E = -\sum_{i \in V} h_i s_i - \sum_{(i,j) \in E} J_{ij} s_i s_j , \qquad s_i \in \{+1, -1\}. \ \text{(Ising model)}$$

2

Introduce classical Hamiltonian $\mathcal{H}_{\mathrm{c}}$ and quantum Hamiltonian $\mathcal{H}_{\mathrm{q}}$.

$$\mathcal{H}_{\mathrm{c}} = -\sum_{i \in V} h_i \sigma_i^z - \sum_{(i,j) \in E} J_{ij} \sigma_i^z \sigma_j^z , \qquad \sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\mathcal{H}_{\mathrm{q}} = -\sum_{i \in V} \sigma_i^z , \qquad \sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

3

Total Hamiltonian is constructed.

$$\mathcal{H}(t) = \mathcal{H}_{\mathrm{c}} + \Gamma(t) \mathcal{H}_{\mathrm{q}}$$

Proposed method

4

Mean-field approximation (decoupling approximation)

See next page for details

$$\rho = \frac{1}{Z}\exp\left(-\frac{\mathcal{H}}{T}\right) \rightarrow \rho = \otimes_i \rho_i$$

K. Tanaka and T. Horiguchi, Scripta Technica, Electron Comm Jpn Pt 3, 83(3): 84–94, 2000

Density matrix: $2^n \times 2^n$ matrix

Local density matrix: 2×2 matrix

$$\rho_i = \frac{1}{Z_i}\sum_{n=1}^{2} |\phi_i(n)> \exp\left(-\frac{1}{T}\lambda_i(n)\right) <\phi_i(n)|$$

$$Z_i \equiv \sum_{n=1}^{2} \exp\left[-\frac{1}{T}\lambda_i(n)\right]$$

Eigenvectors of $\mathcal{H}_i$

Eigenvalues of $\mathcal{H}_i$

$$\mathcal{H}_i = \begin{pmatrix} -h_i - \sum_{j=1,j\neq i}^{N} J_{ij}\langle\sigma_j^z\rangle & -\Gamma(t) \\ -\Gamma(t) & h_i + \sum_{j=1,j\neq i}^{N} J_{ij}\langle\sigma_j^z\rangle \end{pmatrix}$$

5

Decrease temperature and quantum field simultaneously. Go to step 4.
When temperature and quantum field become nearly zero, we obtain the final state.

$$s_i \leftarrow \mathrm{sign}(\langle\sigma_i^z\rangle)$$

Details of step 4

```
temperature = <high temperature>
gamma = <high quantum field>
While temperature > temperature_low or quantum_field > quantum_field_low
    difference = <some value>
    k = 0
    While difference > epsilon(*)
        ⟨σ_i^z⟩_{k+1} = Tr [σ_i^z (ρ_i)_k]
        difference = Σ_{i=1}^{n} |⟨σ_i^z⟩_{k+1} − ⟨σ_i^z⟩_k|
        k ← k+1
    temperature ← temperature * alpha_temperature
    gamma ← gamma * alpha_gamma
```

For $\Gamma = 0$, the equation is the same as the self-consistent equation in the classical Ising model.

$$\langle \sigma_i^z \rangle_{k+1} = \tanh\left(\frac{\sum_{j \neq i} J_{ij} \langle \sigma_j^z \rangle_k + h_i}{T}\right)$$

- The above pseudocode is the case where the state is ***converged*** at the given conditions (temperature and quantum field).
- We compare the above case and another case in which the state is not converged at the given conditions (temperature and quantum field).

Conclusion

Classification of mean-field annealing

	Simulated annealing (SA)	Quantum annealing (QA)	Hybrid quantum annealing (HQA)
Reference	G. Bilbro et al., NIPS1998	K. Tanaka and T. Horiguchi, Scripta Technica, Electron Comm Jpn Pt 3, 83(3): 84–94, 2000	Proposed in this study
Control parameters	Temperature under zero quantum field	Quantum field under zero temperature	Temperature and quantum field

- We consider the performance of HQA of random Ising spin systems comparing with SA and QA.
- When the state is converged in each condition (temperature and/or quantum field), the final state obtained by HQA is almost the same as SA and QA. But the numbers of steps needed to converge are different.
- When the state is not converged in each condition, the final state obtained by HQA differs from that by SA and QA.