Behavior of Analog Quantum Algorithms

Lucas T. Brady

Lucas Kocia, Przemek Bienias, Aniruddha Bapat, Yaroslav Kharkov, Alexey V. Gorshkov

Joint Center for Quantum Information and Computer Science (QuICS)
National Institute of Standards and Technology/University of Maryland

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arXiv soon

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Analog Quantum Algorithms

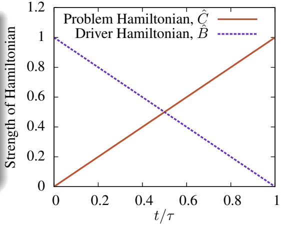
$$\hat{H}(t) = u(t)\hat{B} + (1 - u(t))\hat{C}$$

- $\hat{B}$: Simple Hamiltonian with easy to prepare ground state
- $\hat{C}$: Complicated Hamiltonian with ground state we want to know
- The goal is to transfer from the ground state of $\hat{B}$ to the ground state of $\hat{C}$
- There are multiple design philosophies for how to do this

Quantum Adiabatic Optimization

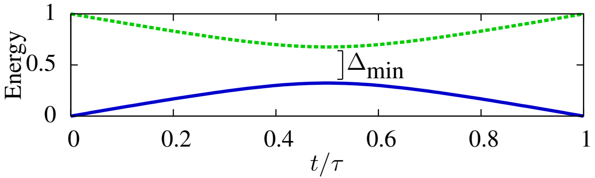
Algorithm

1. Start in the ground state of $\hat{B}$
2. Slowly change the system in total time τ
3. At $t = \tau$, measure to get ground state of $\hat{C}$.



$$\tau_{AC} \gg \frac{\left\|\frac{d\hat{H}}{d(t/\tau)}\right\|}{\Delta_{min}^2}$$

- If this condition holds, adiabaticity is guaranteed*



Quantum Annealing

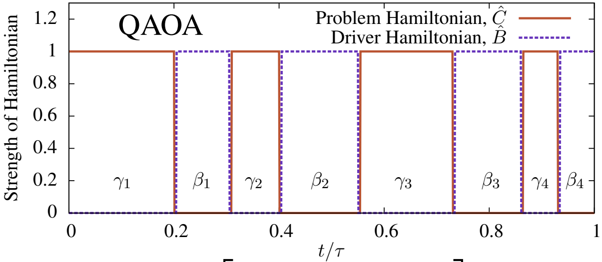
- What happens if you go faster than adiabatic or have a lot of noise?

- Quantum Annealing is non-ideal QAO
- Often there are no/fewer guarantees of success
- Often works partially and justifies the tradeoff of quality for speed
- Leads to weird, complicated dynamics

Google

D:Wave
The Quantum Computing Company™

Quantum Approximate Optimization Algorithm



$$|x(\tau)\rangle = \left[\prod_{j=1}^{p} e^{-i\beta_j \hat{B}} e^{-i\gamma_j \hat{C}}\right] |\varphi\rangle$$

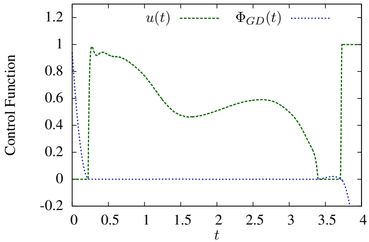
- Treat the quantum computer as a black-box $E(\vec{\gamma}, \vec{\beta}) = \langle x(\tau)|\hat{C}|x(\tau)\rangle$
- Use a classical optimizer to search for the lowest energy by varying γs and βs

Bang-Anneal-Bang

$$\hat{H}(t) = u(t)\hat{B} + (1 - u(t))\hat{C}$$

- We can also ask for the optimal form of $u(t) \in [0, 1]$[1]

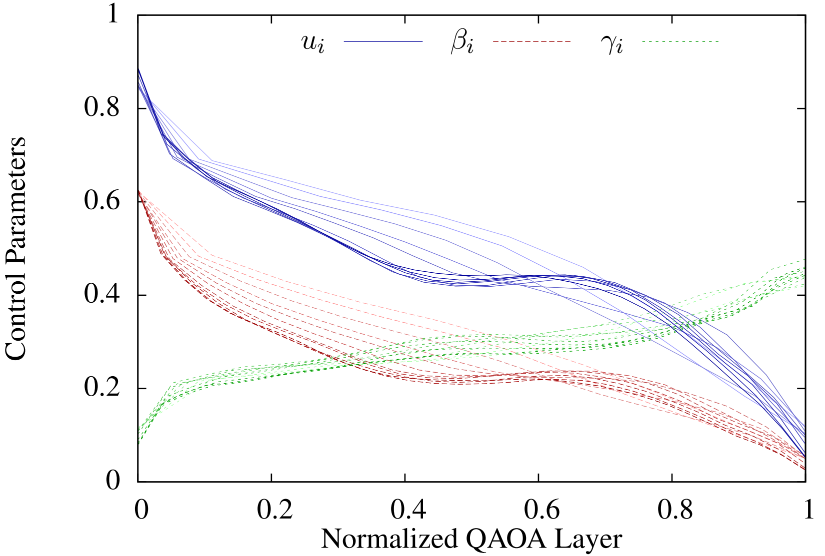
- Optimal procedure has bangs at the beginning and end
- In the middle, there is a smooth annealing region
- The initial and final bangs decrease in length as time increases



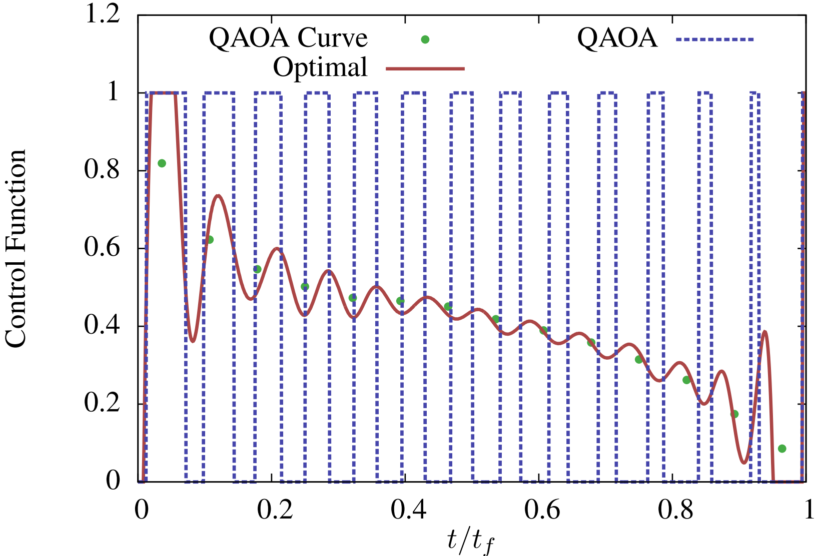
[1] L. T. Brady, C. L. Baldwin, A. Bapat, Y. Kharkov, A. V. Gorshkov, PRL **126**, 070505 (2021)

Numerics

QAOA Curves


u_i
β_i
γ_i
Control Parameters
Normalized QAOA Layer
0
0.2
0.4
0.6
0.8
1

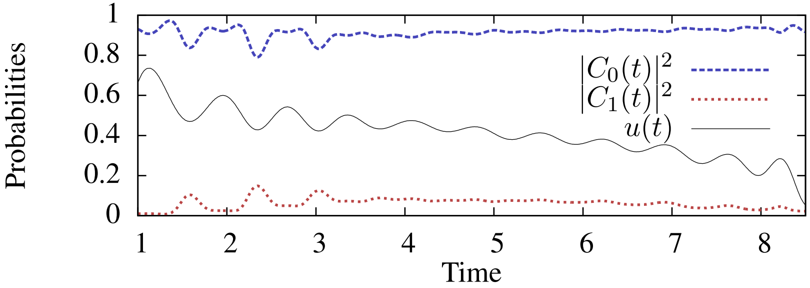
QAOA and BAB


1.2
1
0.8
0.6
0.4
0.2
0
Control Function
QAOA Curve
Optimal
QAOA
0
0.2
0.4
0.6
0.8
1
t/t_f

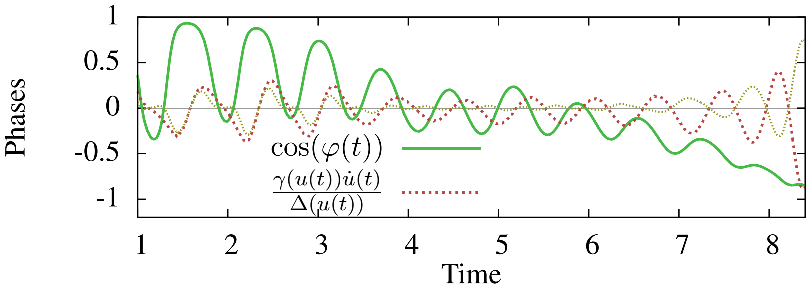
Oscillations

Base Annealing Curve

- As $t_f \to \infty$, the initial and final bangs vanish and the oscillations have smaller amplitude
- The protocol asymptotically becomes adiabatic
- Even non-adiabatically, it maintains instantaneous populations



Oscillations



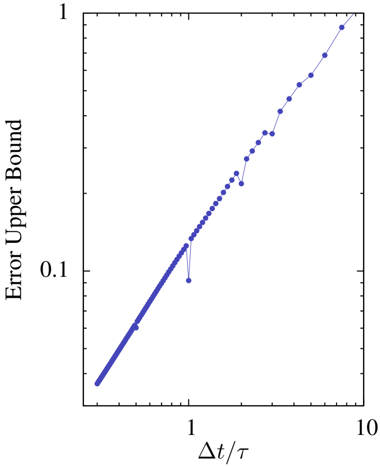
- The oscillations come from the near-adiabatic interaction of the ground state and first excited state
- When the relative phase, $\varphi(t)$, is zero, leakage is small

$$\Theta[u(t)] \equiv \int_0^{t_f} dt \frac{\gamma(u(t))\dot{u}(t)}{\Delta(u(t))} \cos(\varphi(t))$$

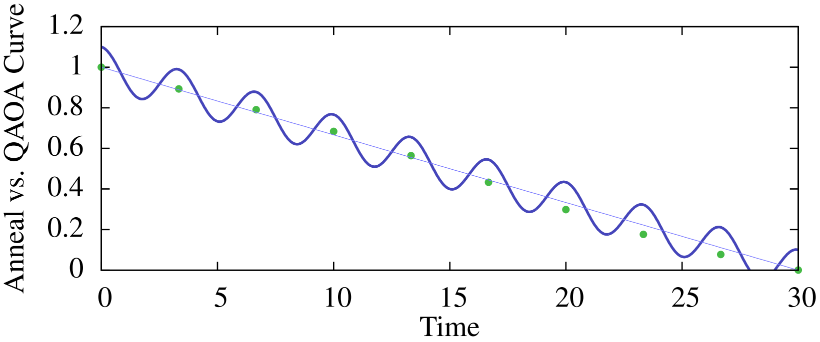
Product Formulas

Enhancing the Product Formula

- QAOA has the form of a product formula
- It is beneficial to match the product formula step to the oscillation period
- The step sizes are still too large
- This works* with Yi & Crosson's new Adiabatic Trotter Error as well (arXiv:2102.12655)



Coherent Oscillations



- QAOA is not a strict product formula
- The optimization can operate over multiple layers, causing coherent error cancellation
- We derive one way for this coherence to occur

Conclusions

- All these algorithms are connected
- QAOA curves are related to optimized adiabatic schedules