Effect of additional operations for constrained quantum annealing

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Quantum Annealing

$$H(s) = sH_{\mathrm{p}} + (1-s)H_{\mathrm{d}}$$

H_{d} : driver Hamiltonian
H_{p} : problem Hamiltonian

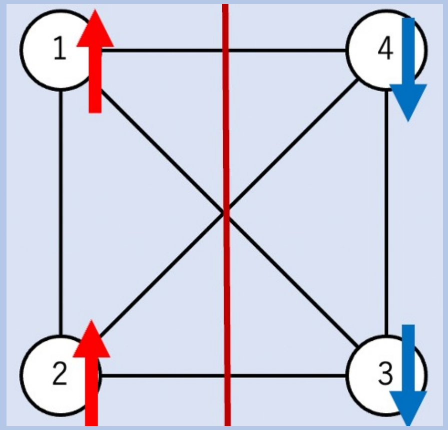
Graph Partitioning

Itay Hen and Federico M. Spedalieri, Phys. Rev. Applied **5**, 034007 (2016).

$$\begin{cases} H_{\mathrm{p}} = \dfrac{1}{2}\displaystyle\sum_{(ij)\in E} W_{ij}(1-\sigma_i^z\sigma_j^z) + \alpha\left(\sum_{i=1}^{n}\sigma_i^z\right)^2 \\ H_{\mathrm{d}} = -\displaystyle\sum_{i=1}^{n}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) \end{cases}$$

Spin reversed transformation

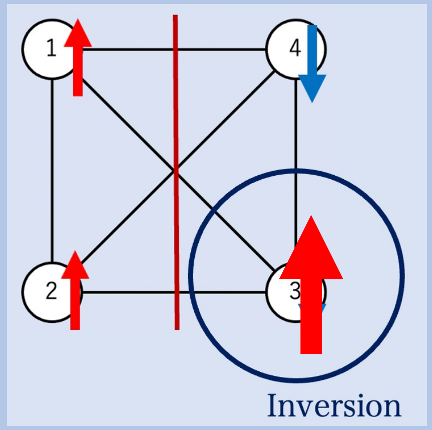
Motivation
When using XX-YY type H_{d}, the performance of quantum annealing is affected by the spin reversal transformation. Then, we consider the effect of spin reversal transformation from the microscopic viewpoint.



One of the solutions of graph partitioning of the complete graph with four vertices

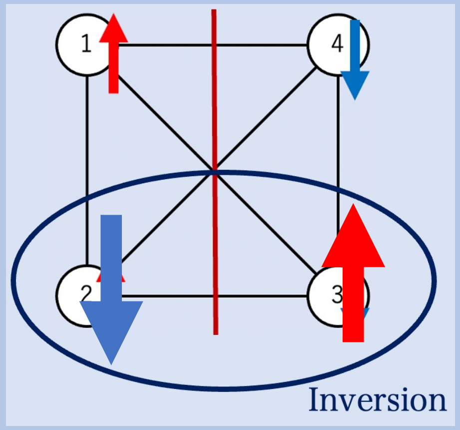
Spin reversal transformation

Example 1



$I \otimes I \otimes \sigma_3^x \otimes I$

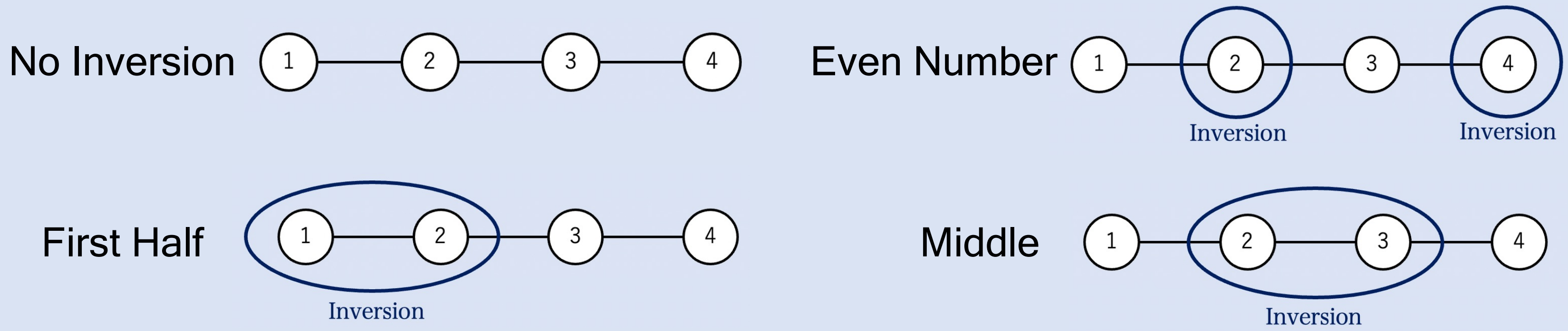
Example 2



$I \otimes \sigma_2^x \otimes \sigma_3^x \otimes I$

Simulation

Pattern of transformations
4 ways



×

The strength of penalty term
$\alpha = 0.5, 1.0, 5.0, 10.0$
4 ways

×

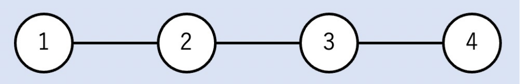
Annealing Time
$\tau = 0.01, 0.1, 0.5, 1.0$
4 ways

64 ways

×

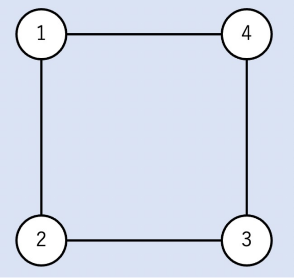
chain (OBC)

$$H_\mathrm{d} = -\sum_{i=1}^{n-1}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y)$$



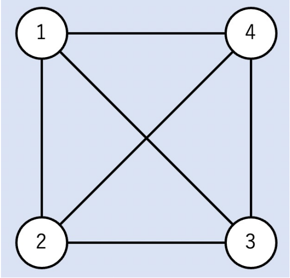
chain (PBC)

$$H_\mathrm{d} = -\sum_{i=1}^{n}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y)$$



complete graph

$$H_\mathrm{d} = -\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right)$$

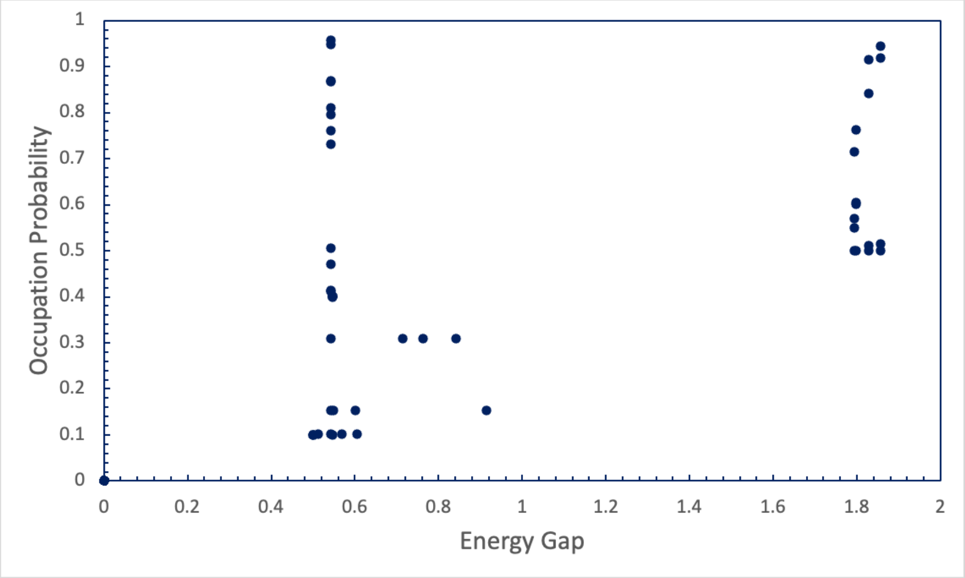


- Chain with open boundary condition (OBC)
  - H_d on chain (OBC)
  - H_d on complete graph
- Chain with periodic boundary condition (PBC)
  - H_d on chain (PBC)
  - H_d on complete graph
- Complete Graph — H_d on complete graph

Results

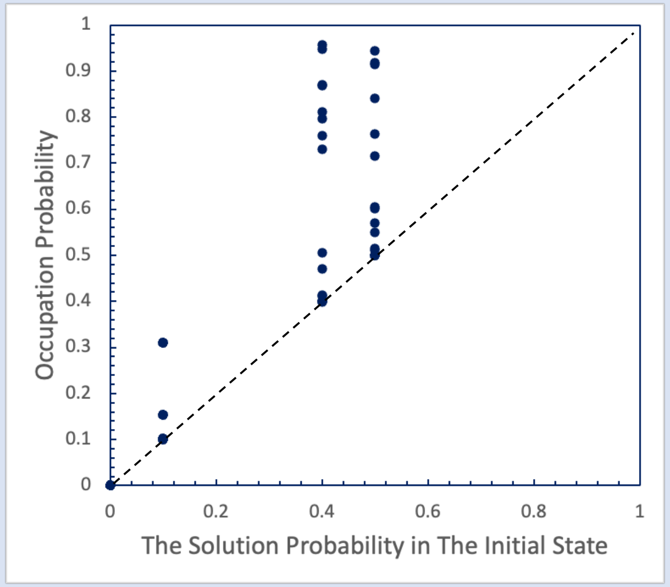
H_p : chain (OBC), H_d : chain (OBC), 4 spins

The relation between **the minimum energy gap** and the occupation probability in the ground state



As the minimum energy gap is higher, the occupation probability is also higher.

The relation between **the solution probability in the initial state** and the occupation probability in the ground state



As the solution probability in the initial state is higher, the occupation probability is also higher.

The above results are obtained by QuTiP

J. R. Johansson, P. D. Nation, and F. Nori, Comp. Phys. Comm. **183**, 1760-1772 (2012)
J. R. Johansson, P. D. Nation, and F. Nori, Comp. Phys. Comm. **184**, 1234 (2013)