A Spin Fixation Method for the Balanced Pick-up Service Problem Using an Ising machine

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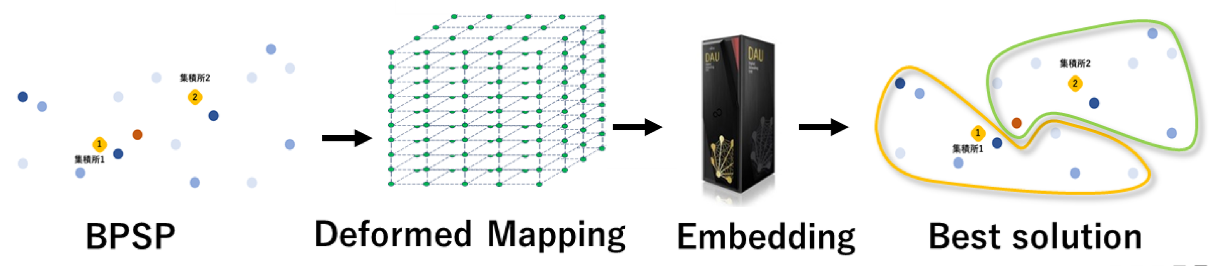
Introduction

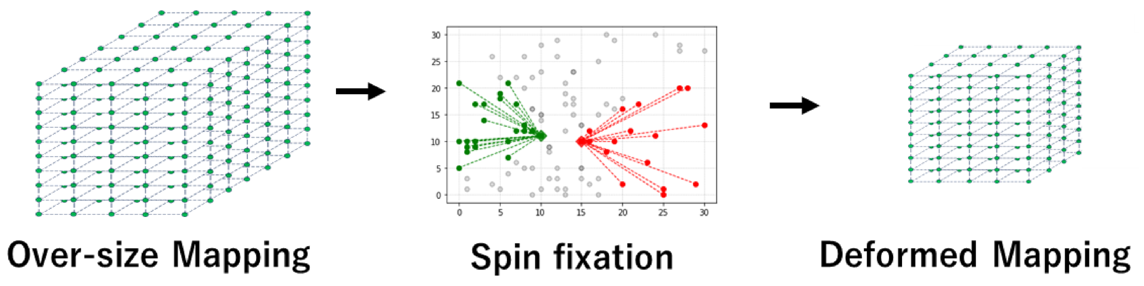
◆Balanced Pick-up Service Problem (BPSP)

The balanced pick-up service problem (BPSP) is an optimization problem concerned with the optimal selection of depots (e.g. postal offices) to collect customers' items.

◆Solution for the BPSP Using Ising machine:



(a) Main process



(b) Deformed mapping with spin fixation

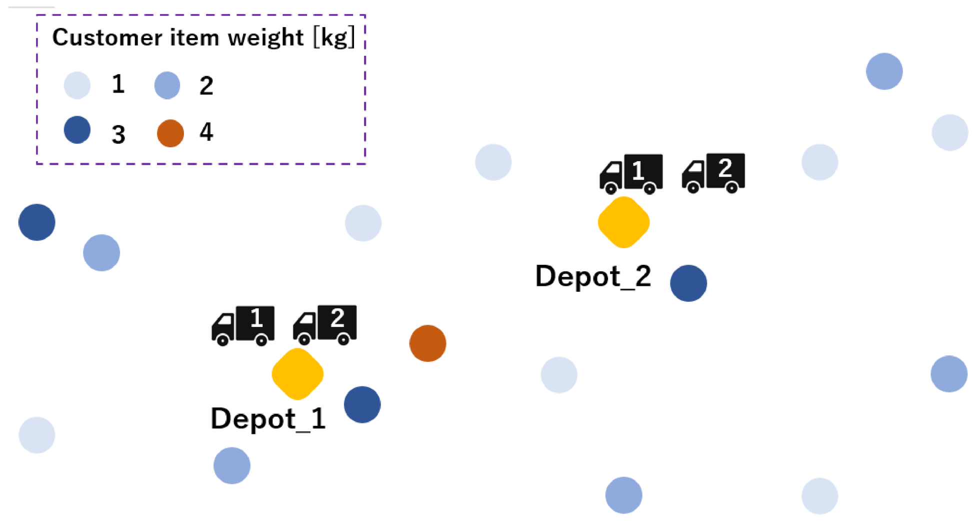
Problem Formulation

◆Input:

(1) Customer $\boldsymbol{m}$ location

(2) Customer weight $\boldsymbol{w_m}$

(3) Depot $\boldsymbol{p}$ location



◆Objectives:

The objective is to minimize two cost functions:

(1) the total distance between the depots and customers

(2) the load variances between depots

◆Constraints:

(1) Each customer must be assigned to exactly one depot.

Problem Mapping with Spin Fixation

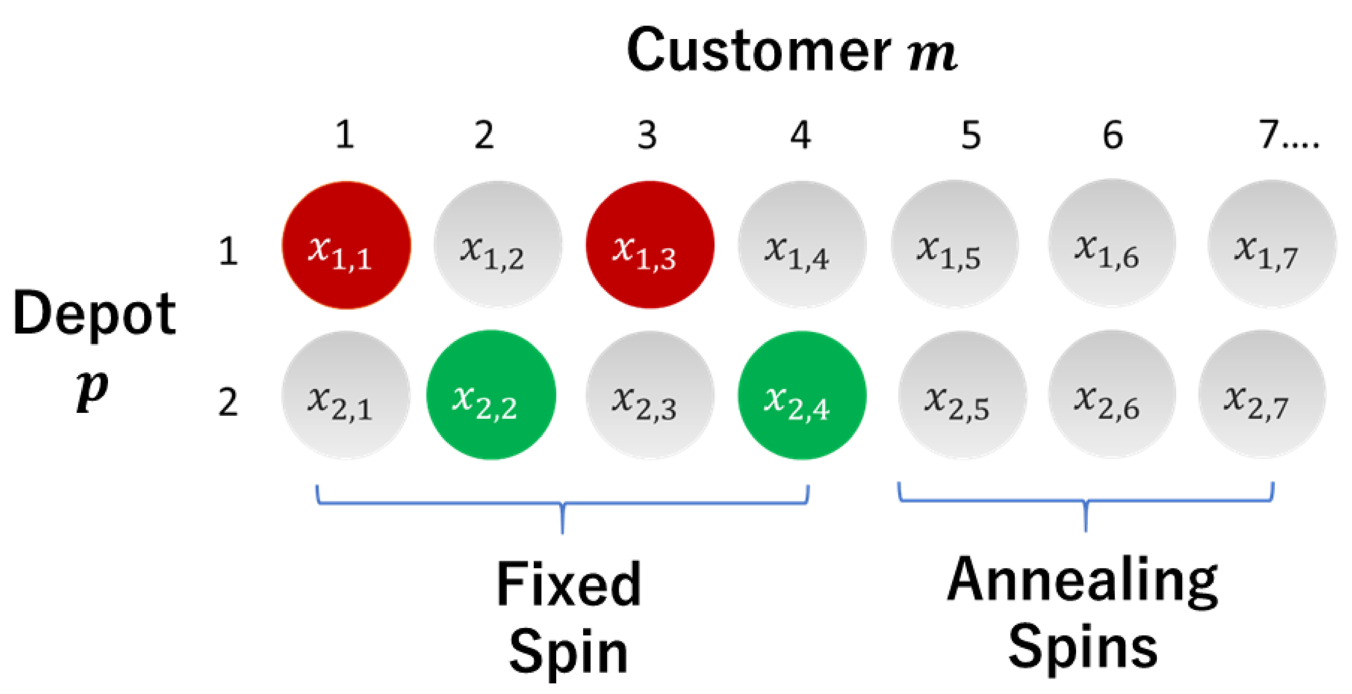
◆Problem mapping:

We map the BPSP onto the clustering problem which is an extension to NP hard the Knapsack Problem (KP) [1].

◆Spin fixation:

Due to the limited number of spins on current Ising machines, we also develop a spin fixation method to reduce the number of spins required by Ising machines.

-> **Several spins (customers) are fixed based on the customer-depot distances c_{pm} .**



◆Ising model:

$$x_m^p = \begin{cases} 1, & \textit{customer m assiged to depot p} \\ 0, & \textit{otherwise} \end{cases}$$

$$\textit{Energy function: } H = \alpha H_A + \beta H_B + \theta H_C$$

$$H_A = \alpha \sum_{m=1}^{M} \left(1 - \sum_{p=1}^{P} x_m^p\right)^2$$

Constraint①: Each customer must be assigned to exactly one depot.

$$H_B = \beta \sum_{p=1}^{P} \sum_{m=1}^{M} c_{pm} x_m^p$$

Objective①: Sum of travel distances between depots and customers should be minimized.

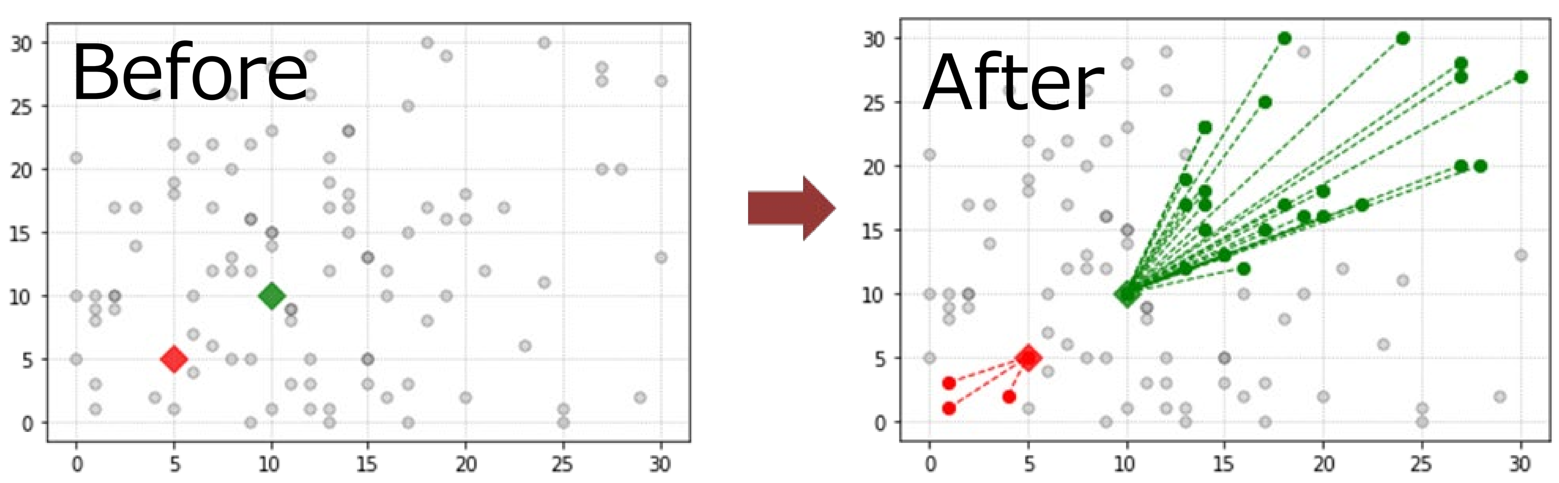
$$H_C = \frac{\theta}{P} \sum_{p=1}^{P} \left(\frac{1}{P} \sum_{p=1}^{P} \sum_{m=1}^{M} w_m x_m^p - \sum_{m=1}^{M} w_m x_m^p \right)^2$$

Objective②: Depot load variance should be minimized.

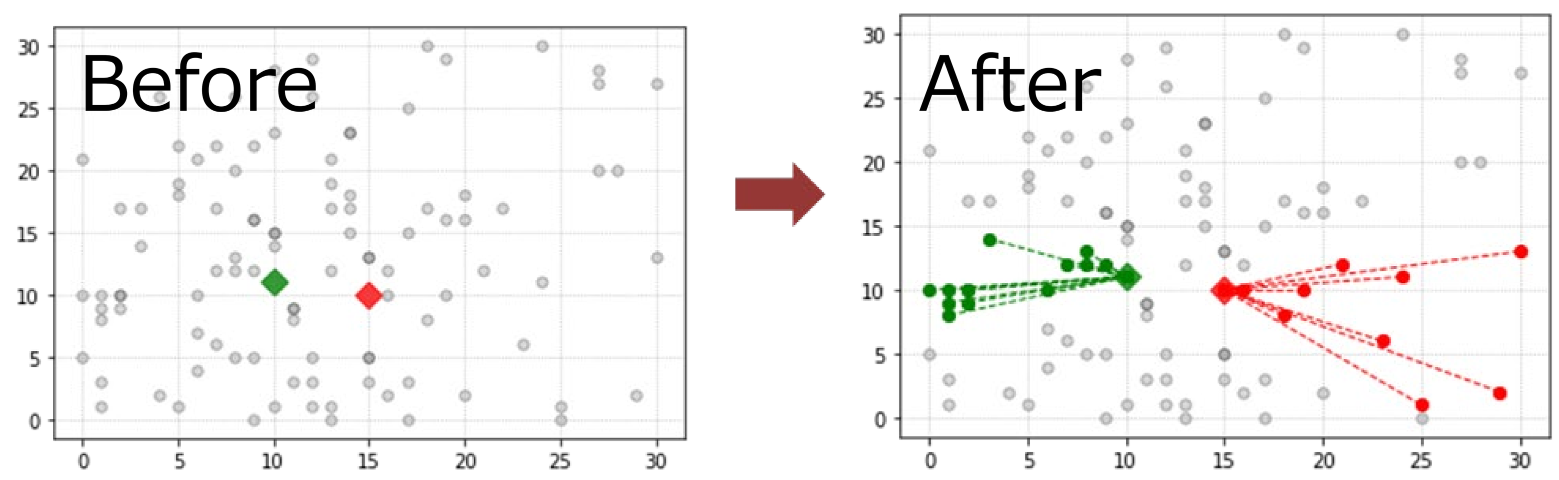
Experimental Results

◆Spin fixation condition:

If the difference **|Dis_depot1-Dis_depot2|>δ**, then assign the customer to its nearest depot. *: δ=max{|Dis_depot1−Dis_depot2|}



Ex① :
Customer =100
Depot =2
δ=7
Fixed customer =28
Fixed spins =56



Ex② :
Customer =100
Depot =2
δ=5
Fixed customer =22
Fixed spins =44

Case *	Fixed Spin	Spin reduction [%]**	H_c [km]	H_D	Constraint satisfaction pro.
0	0	-	102.72	0.27	0.98
1	56	28.0↓	101.59↓	0.13↓	1.00↑
2	44	44.0↓	101.70↓	0.15↓	1.00↑

* : Dataset for case 0~2 is of 100 customers and 2 depots.

**: Computation time are reduced around 5~10% as fewer spins are required.

・Our proposed spin fixation method can not only reduce the number of spins required, but also improve the solution quality and computation time.

◆Acknowledgment

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◆Reference

[1] A.Lucas, "Ising formulations of many NP problems." Frontiers in Physics, 2014.