Scalable algorithm for Capacitated Vehicle Routing Problem Using Ising Machines

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Background

- The number of spins in current Ising machines (summarized in Table 1) is too small to solve real-world problems, caused by the hardware limitation.
- Even if the number of spins increases further in the future, there is no guarantee that the Ising machine will obtain a high-quality approximate solution.

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It may be more effective to partition the large problems into those dealt with into the Ising machine.

Table 1. The summary of Ising machines.

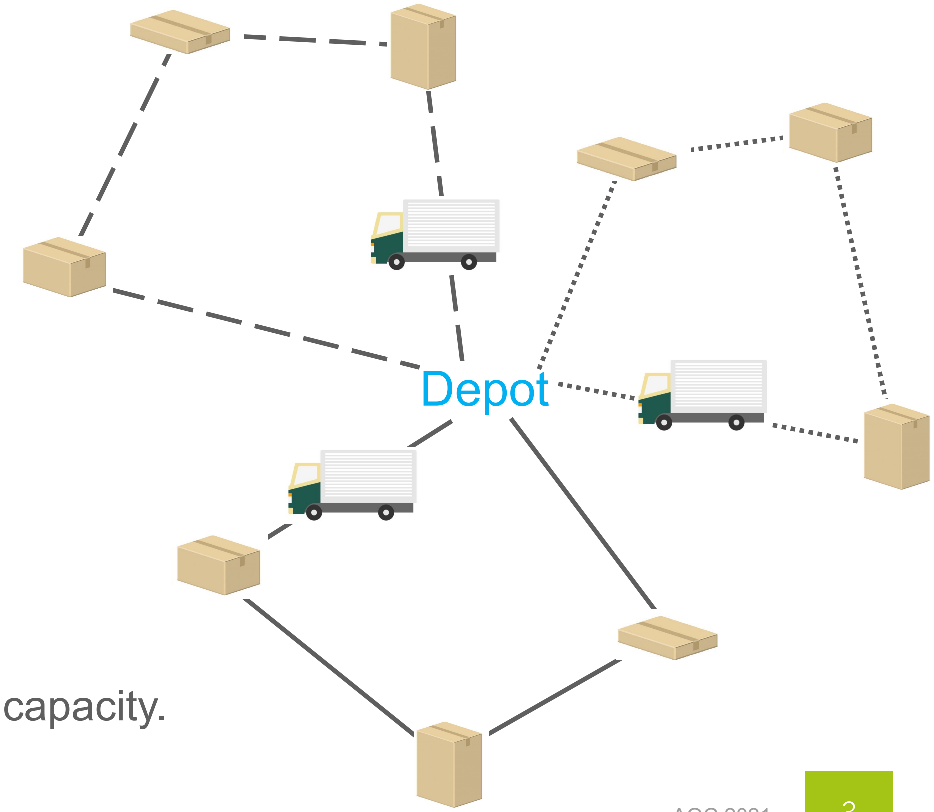
Machine	Topology	#Spins
D-Wave 2000Q [1]	Chimera graph	2048
CMOS Annealing [2]	King's graph	9216
Digital Annealer [3]	Complete graph	8192
CIM [4]	Compete graph	2,000
SBM [5]	Complete graph	100000
Amplify AE [6]	Compete graph	65536

[1] M. Johnson et al., *Nature*, 2011.
[2] M. Hayashi et al., *Symposium on VLSI Circuits*, 2019.
[3] M. Aramon et al., *Frontiers in Physics*, 2019.
[4] T. Okuyama et al., *IEEE ICRC*, 2017.
[5] H. Goto et al., *Science Advances*, 2019.
[6] Fixstars., https://amplify.fixstars.com/.

Capacitated Vehicle Routing Problem (CVRP)

We focus on CVRP as an example of the problem where partitioning can be effectively applied.

CVRP is defined as a problem for finding the optimal route for vehicles to carry out distribution trips starting from the depot to the customer's destination.

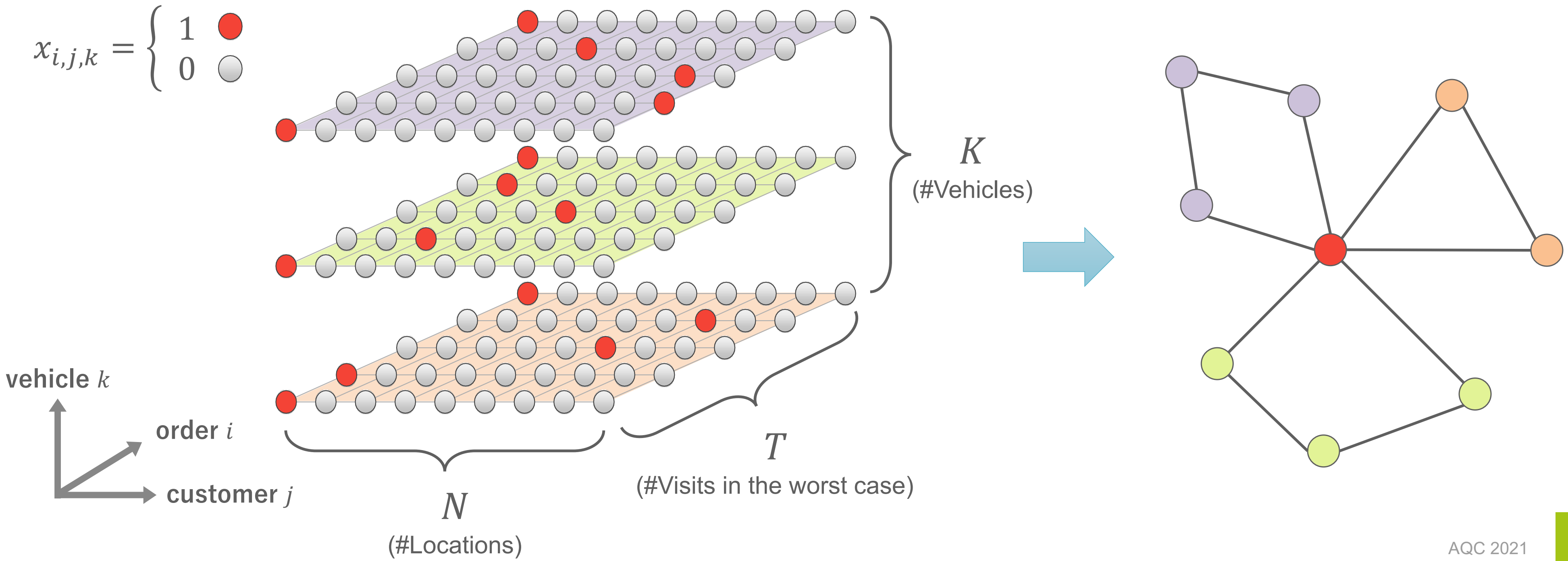


- **Objective:** Minimizing the total length of the routes.
- **Constraints:**
  1. Each route begins and ends at the depot.
  2. Each customer is visited exactly once.
  3. The total demand of each route does not exceed the capacity.

Method uses only Ising machines (w/o partition)

This method is solving the CVRP by a straightforward QUBO formulation.

$x_{i,j,k} \in \{0, 1\}$: if the vehicle k visits the customer j as i-th location on its route.



Method uses only Ising machines (w/o partition)

- Cost $$H_{\mathrm{cost}} = \sum_{k=1}^{K} \sum_{i=0}^{N-1} \sum_{j_1=0}^{N} \sum_{j_2=0}^{N} d_{j_1,j_2} x_{i,j_1,k} x_{i+1,j_2,k}$$
- Constraint1 $$H_{\mathrm{row}} = \sum_{i=1}^{N-1} \left(\sum_{k=1}^{K} \sum_{j=0}^{N} x_{i,j,k} - 1 \right)^2$$
- Constraint2 $$H_{\mathrm{col}} = \sum_{j=1}^{N} \left(\sum_{k=1}^{K} \sum_{i=1}^{T} x_{i,j,k} - 1 \right)^2$$
- Constraint3 $$H_{\mathrm{cap}} = \sum_{k=1}^{K} \left(\sum_{i=1}^{T-1} \sum_{j=1}^{N} \frac{w_j x_{i,j,k}}{Q} \right)^2$$
- Energy function $$H = H_{\mathrm{cost}} + \alpha H_{\mathrm{row}} + \beta H_{\mathrm{col}} + \gamma H_{\mathrm{cap}} \qquad (\alpha, \beta, \gamma > 0)$$

Variable definitions	
N	#Locations
K	#Tracks
T	#Visits in the worst case
Q	Capacity of vehicle
w_j	Demand of customer j
$d_{i,j}$	Distance between two customers i and j

*1: Fix some spins so that the start and end points of each vehicle's route are the depot.

*2: In our experiment, $\alpha = \beta = \gamma = \max d_{i,j}$

Proposed method (w/ partition)

Clustering (Classical computer)

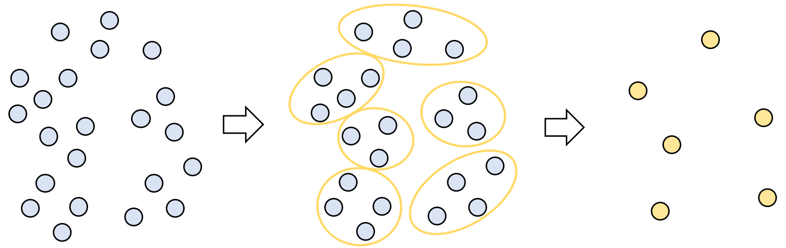
1. The farthest customer from the depot is selected as a cluster seed.
2. The customer which is the closest from the geometric center of the cluster C_j is assigned to C_j among the unassigned customers until the total demand of C_j would exceed the capacity of the vehicles Q.
3. Above processes are repeated until all the customers are assigned to the cluster.

Routing (Ising machine)

1. To find the shortest route within the clusters constructed in the clustering phase.
2. This phase is equivalent to solving the TSP for each cluster with the depot added.
3. If the TSP is too large, apply the scalable algorithm described in the next slide.

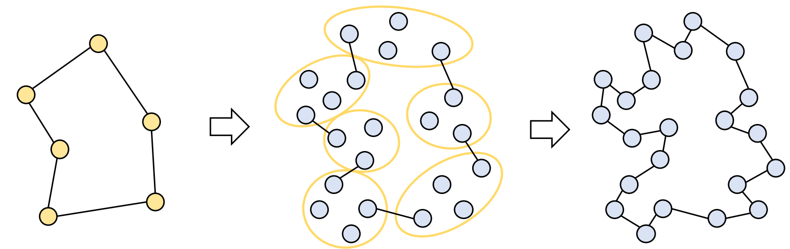
Proposed method (w/ partition)

- **Compression**
  1. Cluster a set of cities. (we use k-means clustering)
  2. Define the geometric center of each cluster as *virtual-city* and construct the new TSP with the virtual-cities.



(a) Compression

- **Decompression**
  1. Solve the compressed TSP using the Ising machine. The obtained route represents the connection of clusters.
  2. In the two clusters to be connected, the edge between the two cities that are most closely to each other is determined in advance.
  3. For each cluster, we solve the TSP whose endpoints are the two cities connected already in previous procedure.



(b) Decompression

*This algorithm is applied recursively until the TSP is scaled down to the size can be dealt with the Ising machine.

Experimental Results

Instance	N	K	#Spins (w/o partition)	Error [%]	
				w/o partition	w/ partition
CMT1	50	5	4845	10.05	12.65
CMT2	75	10	12920	–	10.54
CMT3	100	8	16160	29.48	10.06
CMT4	150	12	25368	42.71	15.87
CMT5	199	16	38400	95.45	12.57
CMT6	50	6	5814	1.85	12.02
CMT7	75	11	14212	22.71	1.54
CMT8	100	9	17271	23.45	5.12
CMT9	150	14	25368	–	2.51
CMT10	199	18	43200	120.42	4.18
CMT11	120	7	23716	55.62	3.13
CMT12	100	10	20200	82.35	7.74
CMT13	120	11	31944	62.41	2.56
CMT14	100	11	22220	47.24	1.89

- Used Ising machine is [6] which has 65,546 spins with complete graph.
- Instances are picked from [7]
- Error [%] means the relative error (avg. of 50 trials) from the best-known solution (BKS).
  "–" is the feasible solution cannot be obtained.
- Except the small problems (CMT1, CMT6), our proposal (w/ partition) can obtain the higher quality solutions than w/o partition.

[6] Fixstars, https://amplify.fixstars.com [7] CVRPLIB, http://vrp.atd-lab.inf.puc-rio.br/index.php/en/

Conclusion

- Our proposed algorithm can obtain feasible solutions for large CVRP that cannot be dealt with to an actual Ising machine.

- In comparison with the straightforward QUBO by Ising machines (w/o partition), our proposal (w/ partition) can obtain the higher quality solutions.

Future Work

- Comparison of the computation time for each method.

- Evaluation by other Ising machines with fewer spins.