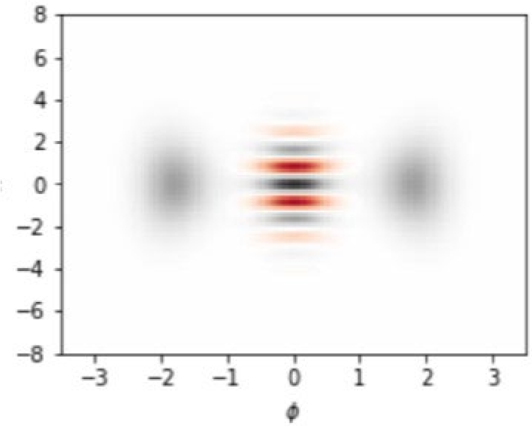
Kirchhoff's Laws as a Classical Model for Superconducting Annealing: Successes and Opportunities



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- Northrop Grumman Colleagues
- QEO/QAFS Collaborators, especially Lincoln Laboratory and University of Southern California
- IARPA and DARPA
- AQC Community

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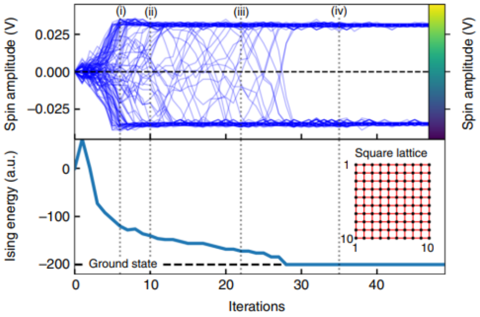
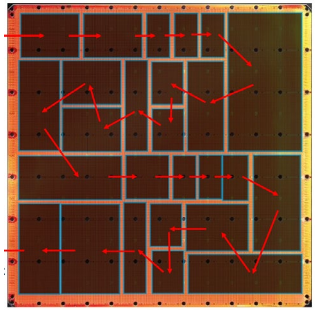
The Slowing of Traditional Computing Scaling Metrics and New Machine Learning Workloads Open an Exciting Era of Novel Computing Architectures

Coherent Ising machines



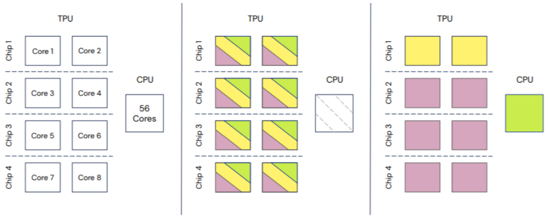
Böhm et al., Nat Comm 10, 3538 (2019)

Wafer Scale Computing



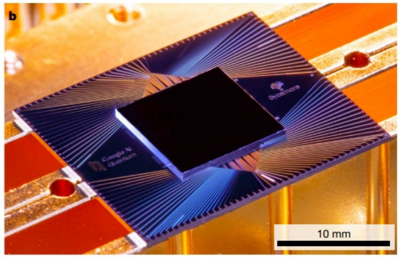
Cerebras Systems, IEEE Hot Chips 32 Symposium (HCS), 2020

TPU Pods



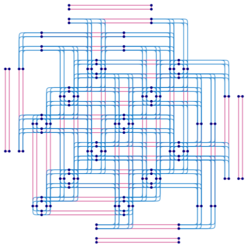
Hessel et al,
arXiv:2104.06272 (2021)

Google Sycamore Processor



Arute et al, Nature 574, 505 (2019)

D-Wave Computer



Boothby et al, arXiv:2003.00133 (2020)

DARPA's Quantum Annealing Testbed

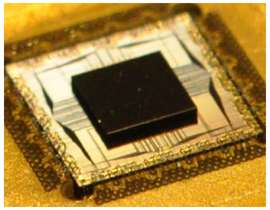


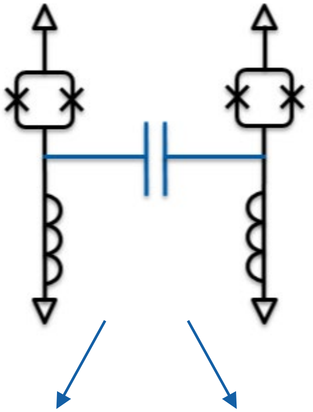
Image courtesy of MIT
Lincoln Laboratory

Superconducting Annealing Computing, **Both Quantum and Classical**, are Promising Compute Frameworks

The Choice of Abstraction Level can be Important for Understanding the Novel Computational Resource

Case Study: Non-Stoquasticity

Is this Non-Stoquastic?



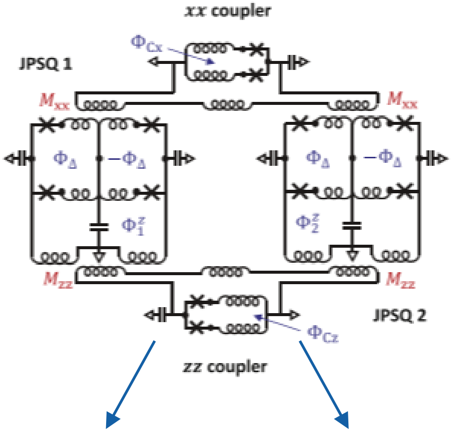
Two Level Description

Yes?

Circuit Hamiltonian

No[2,3]

Is this Non-Stoquastic?[4]



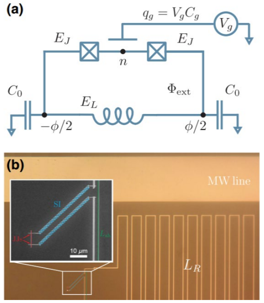
Two Level Description

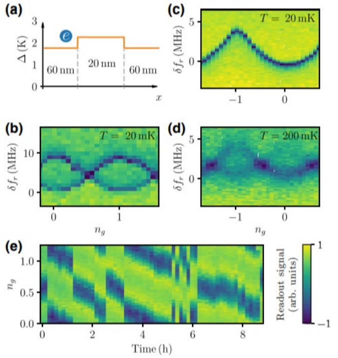
Yes

Circuit Hamiltonian

Yes

Can You Make It?[5,6]

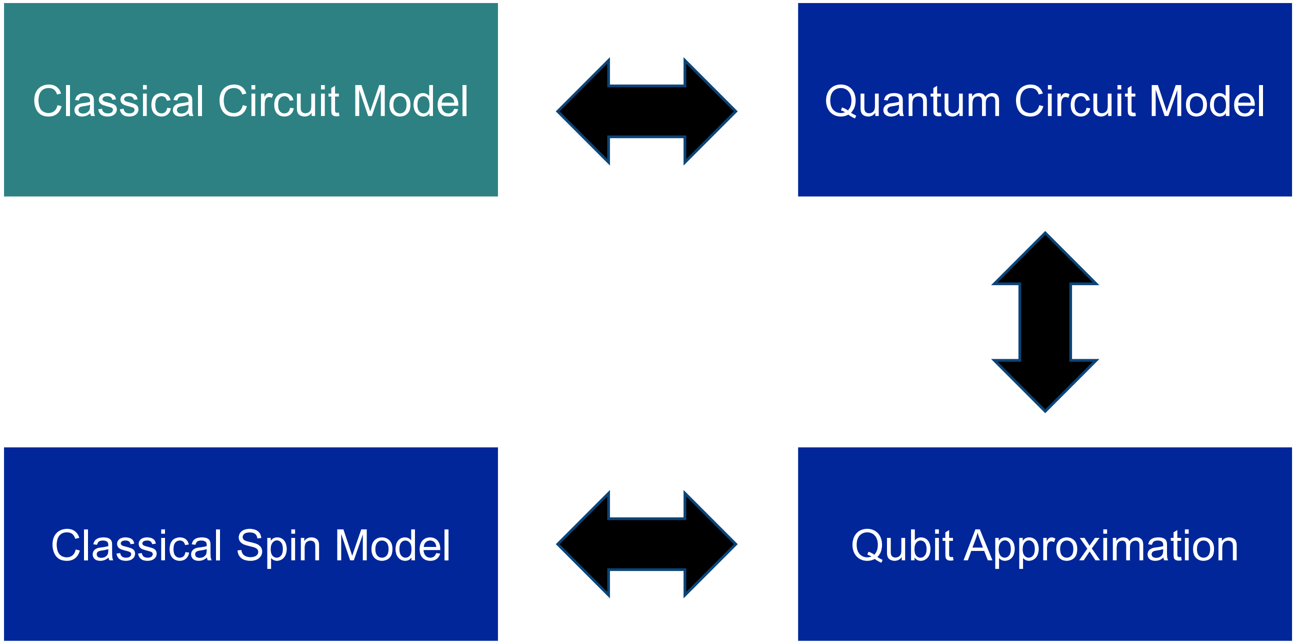




Yes

1. Ozfidan et al. PR Appl 13, 034037 (2020)
2. Halverson et al., arXiv:2011.03831 (2020)
3. Ciani and Terhal, PRA 103, 042401 (2021)
4. A J Kerman, New J. Phys. 21, 073030 (2019)
5. Hinkey et al., APS March Meeting C29.00005 (2019)
6. Kalashnikov, et al., PRX Quantum 1, 010307 (2020)

The Classical Circuit Model is an Important Model for Understanding Superconducting Annealing


Classical Circuit Model

Quantum Circuit Model

Classical Spin Model

Qubit Approximation

The Classical Circuit Hamiltonian and Quantum Circuit Hamiltonian are Weyl–Wigner Transform Pairs Emphasizing the Naturalness of the Classical Model

Quantum Hamiltonian of Charges and Fluxes

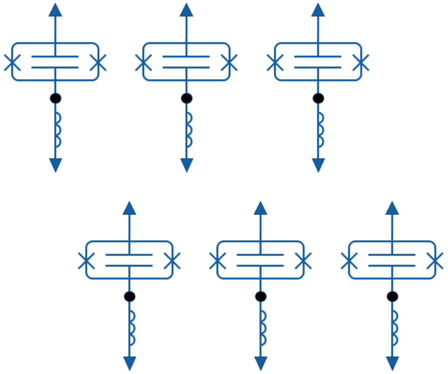
$$\hat{H}_Q = \left(\sum_i \frac{1}{2C}\hat{Q}_i^2 + \sum_i \left(\frac{1}{2L}\left(\hat{\Phi}_i - \Phi_0 z_i\right)^2 - \frac{\beta}{L}\left(\frac{\Phi_0}{2\pi}\right)^2 \cos \pi x_i \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}_\mathrm{i}\right)\right) - \frac{1}{2L}\sum_{ij}\gamma_{ij}\hat{\Phi}_i\hat{\Phi}_j\right)$$

Wigner map

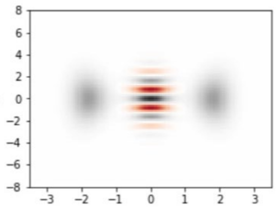
Weyl quantization

Classical Hamiltonian of Charges and Fluxes

$$H_C = \left(\sum_i \frac{1}{2C}Q_i^2 + \sum_i \left(\frac{1}{2L}\left(\Phi_i - \Phi_0 z_i\right)^2 - \frac{\beta}{L}\left(\frac{\Phi_0}{2\pi}\right)^2 \cos \pi x_i \cos\left(2\pi\frac{\Phi_\mathrm{i}}{\Phi_0}\right)\right) - \frac{1}{2L}\sum_{ij}\gamma_{ij}\Phi_i\Phi_j\right)$$



Double Well $|+\rangle$ Wigner Function



In analogy to Hermite polynomials, generalized adiabatic eigenstate Pauli operators are natural basis for density matrices and classical probability distributions

$$\hat{\rho}(t) = \sum_{\vec{\mu}} \rho_{\vec{\mu}}(t)\,\hat{P}_{\vec{\mu}} \qquad P(\Phi, Q, t) = \sum_{\vec{\mu}} f_{\vec{\mu}}\,(t) W_{\vec{\mu}}(\Phi, Q)$$

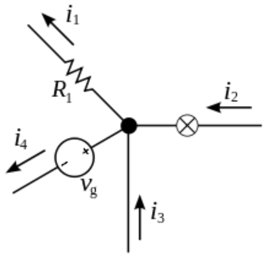
Wigner

Weyl

Hamilton's Equations for the Classical Hamiltonian are Equivalent to Kirchhoff's Circuit Law

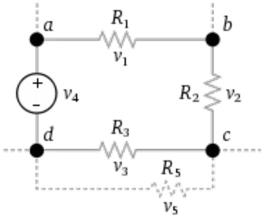
$$\dot{Q}_i = -\frac{\partial H}{\partial \Phi_i} = \sum_e I_{ei}$$

The charge building up on node i (i.e., the current flowing to the capacitors) is equal to the current flowing into the node via the attached circuit elements e



$$\dot{\Phi}_i = \frac{\partial H}{\partial Q_i} = \sum_j C_{ij}^{-1} Q_j$$

The voltage on node i is generated by capacitive interactions with charge on the other nodes j such that it is a potential function

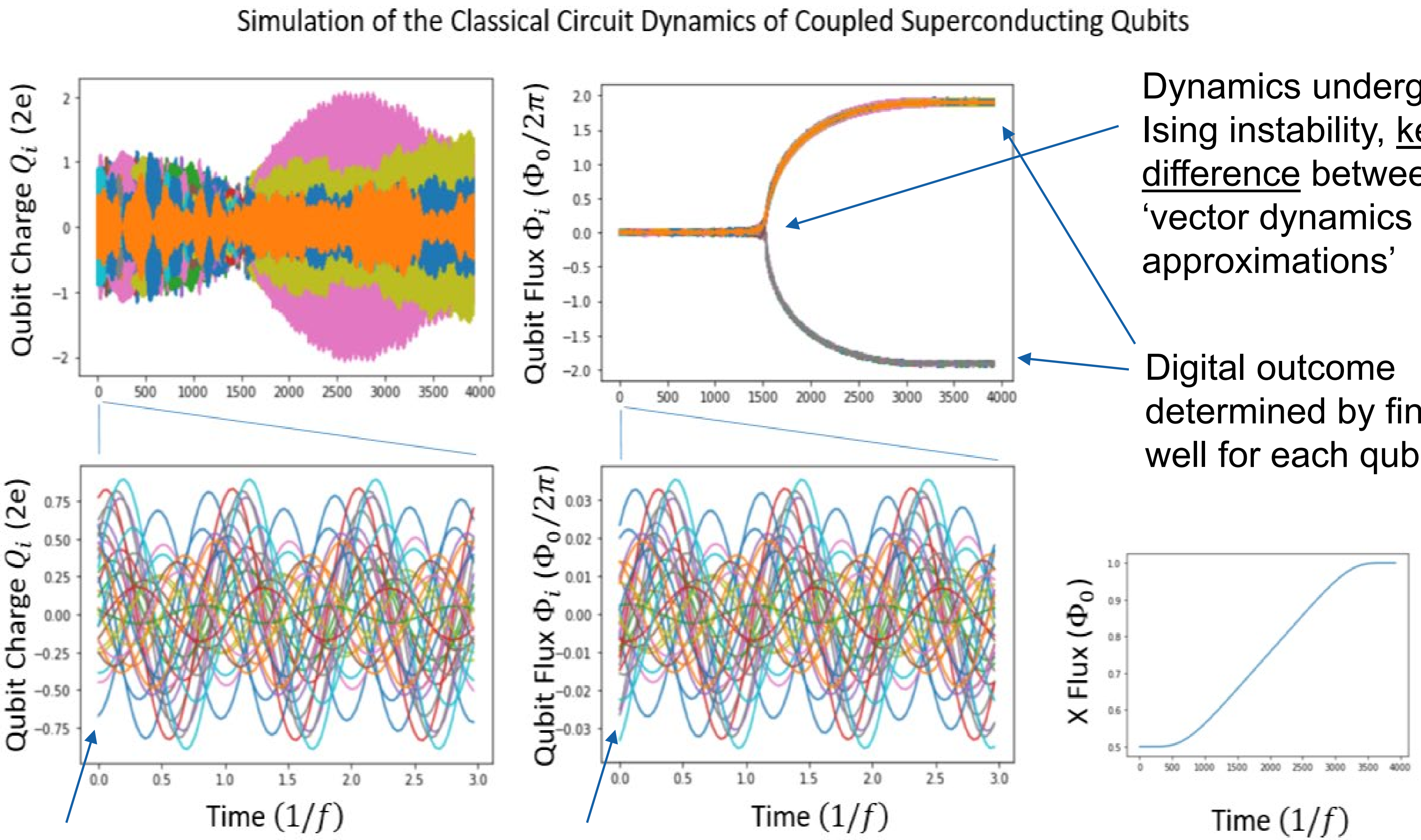


https://en.wikipedia.org/wiki/Kirchhoff%27s_circuit_laws

- Although there are many classical models for annealing the use of classical Kirchhoff's laws is an important benchmark
- Classical superconducting engineers have developed extensive SPICE tools for classical circuit simulation including WRspice[1]
- While there is an extensive overhead (i.e. μs anneals can take seconds to simulate), the time and memory scaling of classical circuit simulations scale linearly in the number of degrees of freedom, though there can be bottlenecks resulting from the simulation hardware

1. http://wrcad.com/wrspice.html

Classical Circuit Dynamics Undergoes an Ising Instability During an Anneal Which Is a Key Distinction with Other Classical Models



Dynamics undergo Ising instability, key difference between 'vector dynamics approximations'

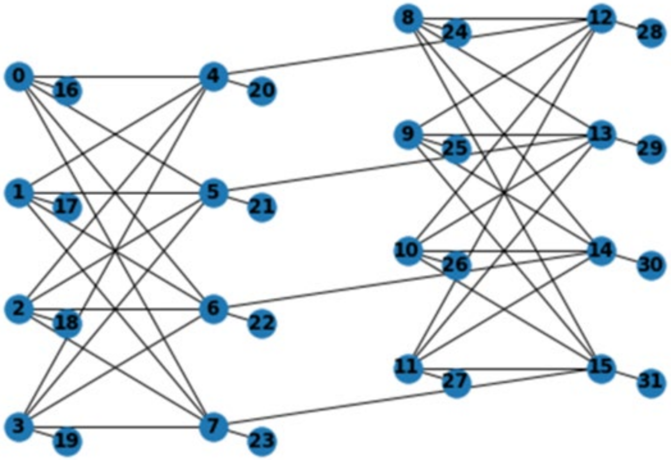
Digital outcome determined by final well for each qubit

Initial conditions sampled from classical thermal distribution

Initial Tests of Classical Circuit Simulation Method Were Implemented for the Boixo et al. Example Circuit Using Auxiliary Variables for External Fields

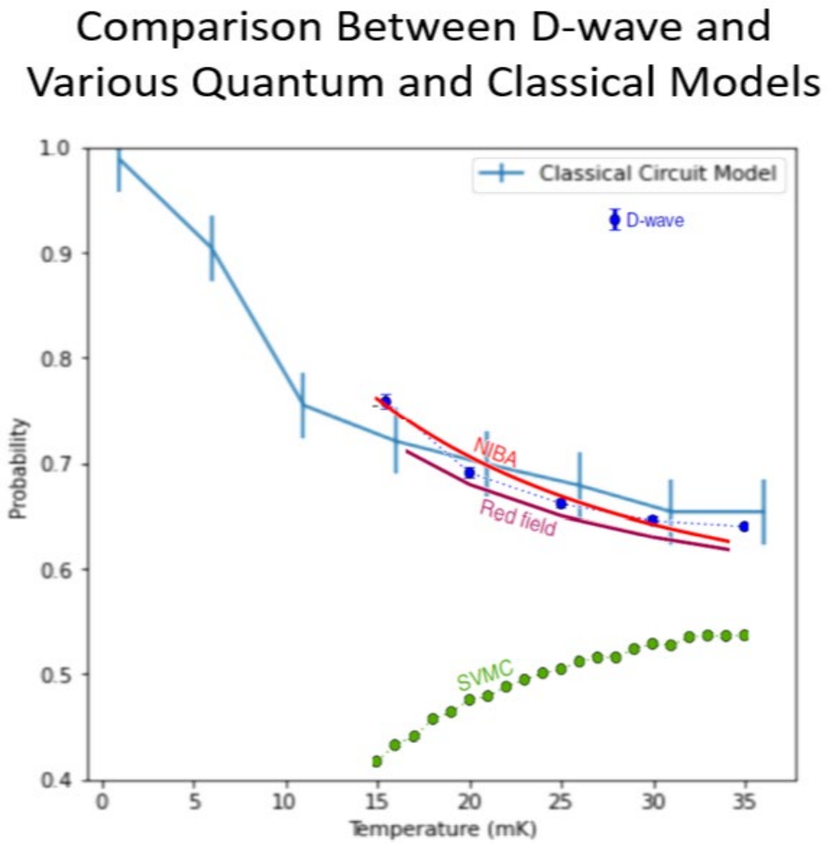
- The classical variables go through an Ising instability driven by the $\cos(\pi x(t))$ term becoming larger than the harmonic restoring force, thus each of the $\phi_i(t)$ begin to grow in magnitude.
- The $z_i(t)\phi_i(t)$ terms should grow to stay in balance with $\phi_i(t)\phi_j(t)$ so that the single qubit terms and the two qubit terms grow proportionally
- One solution is to assign to each qubit an auxiliary qubit whose only role is to provide the external magnetic field to the target qubit.

Graph of Qubit Connections from Boxio et al.[1] with Auxiliary Qubits to Implement Magnetic Field for Qubits



1. Boixo et al. Nat. Comm. 7, 10327 (2016)

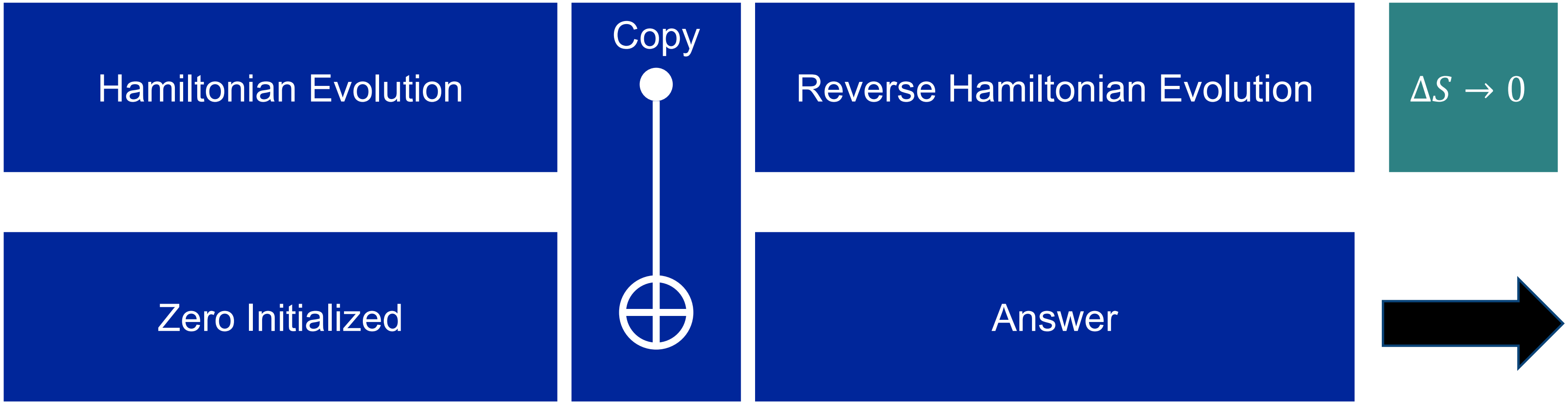
Success: Initial Comparison of Classical Circuit Dynamics Compares Well to D-Wave Experiment



Comparison Between D-wave and Various Quantum and Classical Models

- Unlike SVMC, classical circuit model approaches 100% success probability at low temperatures
- Future directions include studying the effects of simulation details such as annealing ramp rate and correlated initial thermal distribution

Opportunity: Analog Hamiltonian Evolution is Compatible with Zero Entropy Computing[1] Allowing a Potential Power Performance Enhancement



- The flux control waveforms for the reverse Hamiltonian Evolution need to be the time reverse of the forward evolution, and the initial condition of the reverse evolution needs to be the charge conjugation of the final state of the forward evolution
- If ‘reversible’ compute technology can halve their required power consumption, e.g., every two years, over a sustained period this will generate an **exponentially growing advantage** relative to standard digital technology which is intrinsically limited by Landauer's principle[2]

1. C. H. Bennett, IBM Journal of R&D 17, 525 (1973)
2. Landauer, IBM Journal of R&D 5, 183 (1961)

Opportunity: Mermin Inequality[1] Can Lead to Exponential Quantum Advantage for Computational Transition Rates over What is Possible Classically

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Classical Transition Rate for Pauli Wigner Basis Coefficients

$$\dot{f}_\mu = TrH\{W_\mu, W_\nu\}f_\nu$$

Quantum Transition Rates for Density Matrix Pauli Coefficients

$$\dot{\rho}_\mu = \frac{i}{\hbar}\,\hbar\,Tr\widehat{H}\left[\widehat{P}_\mu, \widehat{P}_\nu\right]\rho_\nu$$

Mermin Inequality

For high Hamming weight distance Hamiltonian terms, e.g.:

$$\widehat{M} = |0000\rangle\langle 1111| + |1111\rangle\langle 0000|$$

The expectation value of the certain quantum states e.g.:

$$\hat{\rho}_{GHZ} = \frac{1}{2}|0000\rangle\langle 0000| + \frac{1}{2}e^{-\gamma_\phi t}|0000\rangle\langle 1111| + \frac{1}{2}e^{-\gamma_\phi t}|1111\rangle\langle 0000| + \frac{1}{2}|1111\rangle\langle 1111|$$

can be exponentially larger as a function of Hamming weight distance than is possible classically.
However, this advantage is reduced by dephasing.

1. N. D. Mermin, PRL 65, 1838 (1990)

See You Next Year. In Person!

Superconducting Annealing Computing, **Both Quantum and Classical**, are Promising Compute Frameworks For Next Generation Computers

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