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Extended Ising Machine for Future DAUs

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Challenges of Ising Machines

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- Energy to be minimized is limited to binary quadratic forms
  - Cannot handle more general functional forms (e.g., higher-order terms)
    - Number of terms explodes when higher order terms are converted to quadratic
- Some problems are poorly solved even when converted to quadratic form
  - Difficult to solve when assignment or permutation variables are used
  - Difficult to solve when inequality constraints are formulated using slack variables

We need a scheme to solve the above issues with a highly parallelizable architecture.

DA: Revival of Difference Engine from the 19th Century

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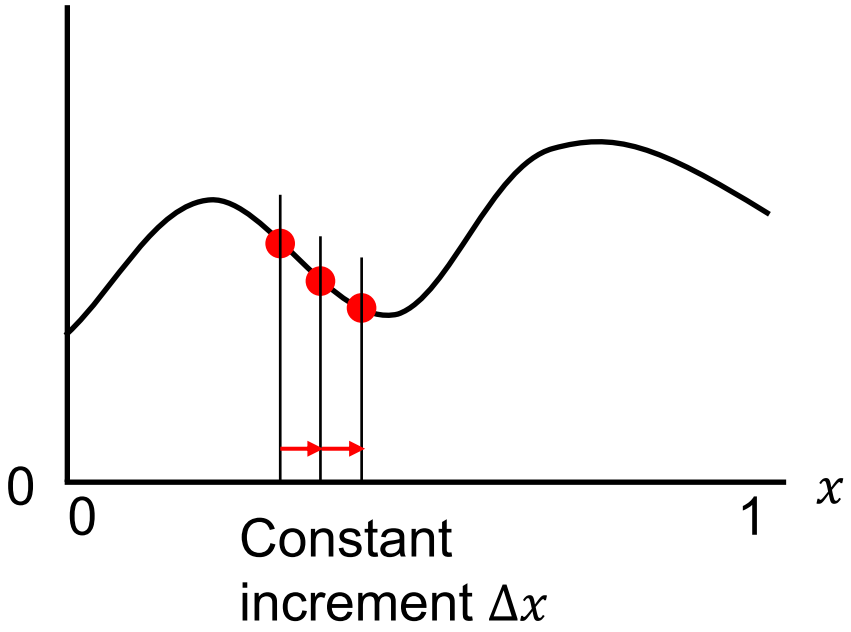
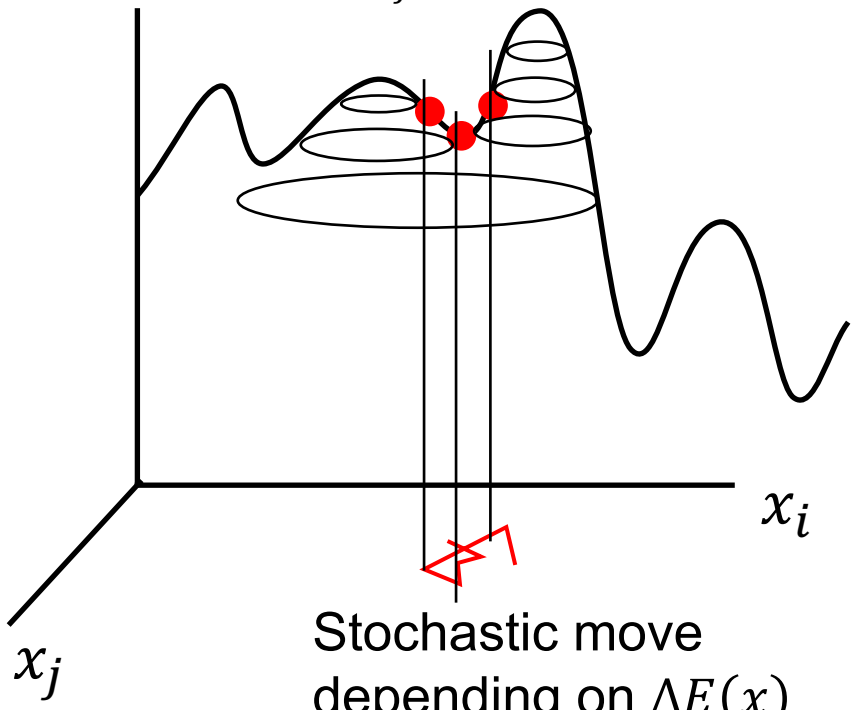
Difference Engine

$$f(x) = \sum_{i=0}^{N} a_i x^n$$



DAU

$$E(x) = -\sum_{i=1}^{N}\sum_{j>i}^{N} W_{ij}x_i x_j - \sum_{i=1}^{N} b_i x_i$$



All You Need is Difference*

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Finite differences

Energy E	1st difference	2nd difference
$E = -\sum_{i=1}^{N}\sum_{j>i}^{N} W_{ij}\, x_i x_j - \sum_{i=1}^{N} b_i x_i$	$h_i = \sum_{j=1}^{N} W_{ij}\, x_j + b_i$	W_{ij}

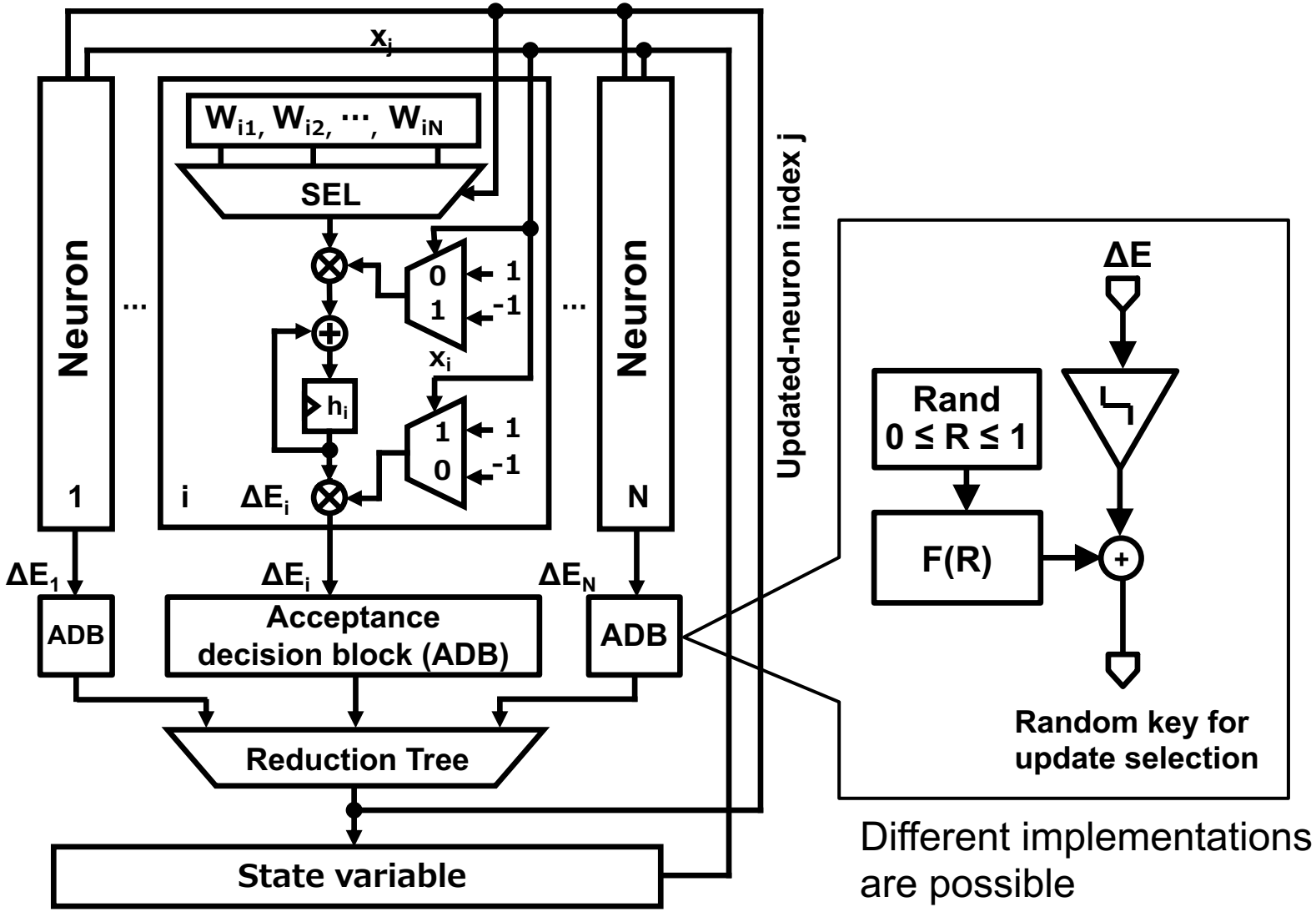
Update scheme when bit x_k is flipped

E	h	x
$E \leftarrow E - h_k \Delta x_k$	$h_i \leftarrow h_i + W_{ik}\, \Delta x_k$	$x_k \leftarrow x_k + \Delta x_k$

Energy increment for parallel trial $\quad \Delta E_i = -h_i(1 - 2x_i)\ for\ i = 1 \cdots N$

*Macoto Kikuchi, "All your need is difference," SF Magazine, 2009, Hayakawa Publishing, Japan

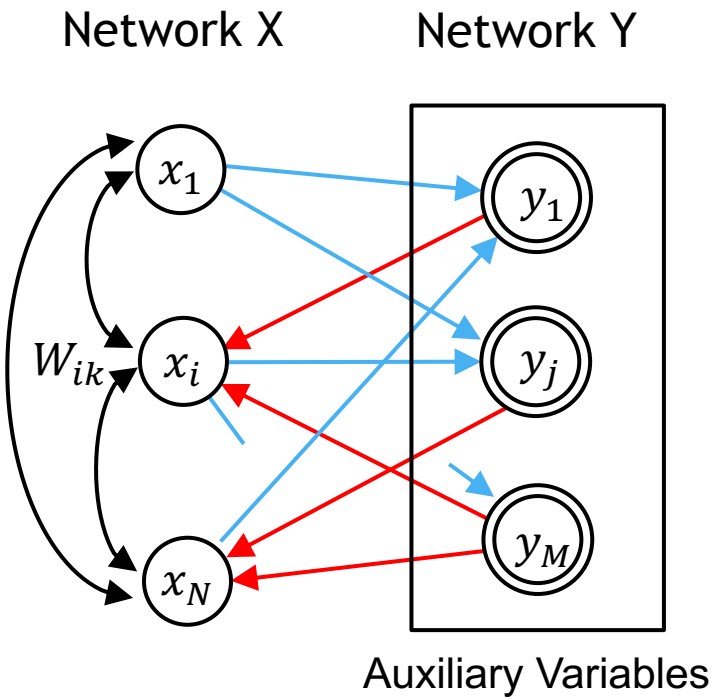
DAU Conceptual Architecture



Different implementations are possible

Generation of Additional Local Fields with Auxiliary Variable

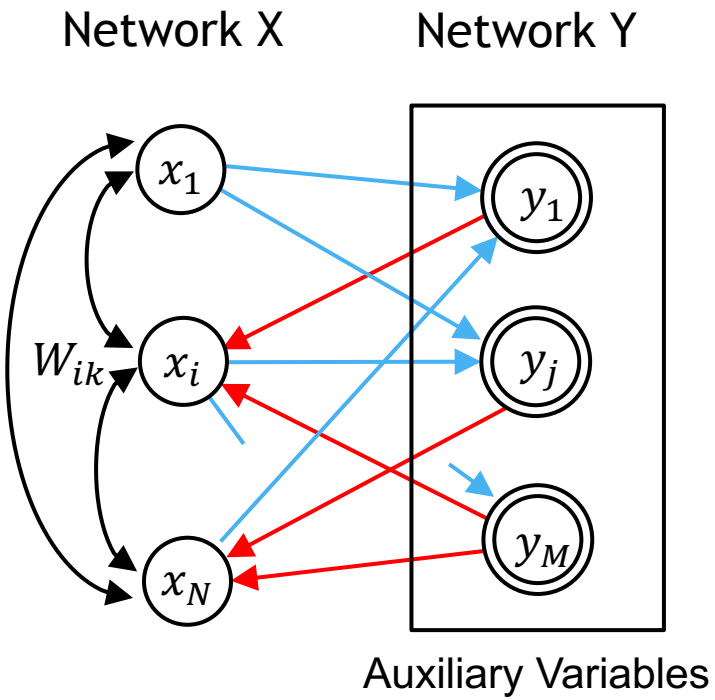
- Embed auxiliary-variable neural nets within the Ising Machine
  - Generate auxiliary variable y in a neural net coupled with decision variable x
  - The local field (restoring force) generated by auxiliary variable y_j acts on variable x_i

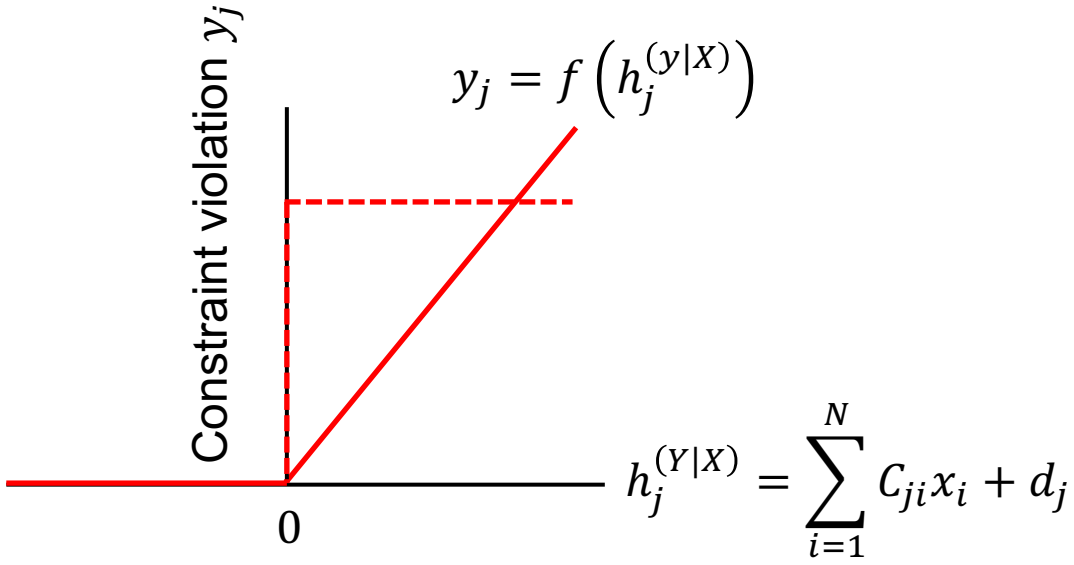


Local field of x_i	$h_i^{(X\|X)} = \sum_{j=1}^{N} W_{ij}x_j + b_i$
Local field of y_i	$h_j^{(Y\|X)} = \sum_{j=1}^{N} C_{ji}x_j + d_j$
Output y_i	$y_j = f\left(h_j^{(y\|X)}\right)$
Additional local field of x_i	$h_i^{(X\|Y)} = \sum_{j=1}^{M} F_{ij}y_j$

Application Example 1: Inequality constraints

- Use a binary neuron whose value is 1 when there is an inequality constraint violation for i-th resource





Simulation Results

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- Pisinger's knapsack problem, hard instances

Intel® Core™ i9-10850K CPU@3.60GHz, CPU cores: 10, cache size: 20408KB

Instance	Result	Iter.	TTS (s)		Instance	Result	Iter	TTS (s)		Instance	Results	Iter.	TTS (s)
knapPI_11_20_1000_1	OK	64	0.028		knapPI_13_20_1000_1	OK	7	0.027		knapPI_15_20_1000_1	OK	11	0.028
knapPI_11_50_1000_1	OK	57	0.065		knapPI_13_50_1000_1	OK	19	0.065		knapPI_15_50_1000_1	OK	20	0.064
knapPI_11_100_1000_1	OK	105	0.125		knapPI_13_100_1000_1	OK	4396	0.126		knapPI_15_100_1000_1	OK	45	0.123
knapPI_11_200_1000_1	OK	118	0.247		knapPI_13_200_1000_1	OK	4339	0.250		knapPI_15_200_1000_1	OK	98	0.250
knapPI_11_500_1000_1	OK	371	0.629		knapPI_13_500_1000_1	OK	4844	0.643		knapPI_15_500_1000_1	OK	235	0.639
knapPI_11_1000_1000_1	OK	489	1.294		knapPI_13_1000_1000_1	OK	1182	1.345		knapPI_15_1000_1000_1	OK	512	1.340
knapPI_11_2000_1000_1	OK	994	2.832		knapPI_13_2000_1000_1	OK	1394	2.951		knapPI_15_2000_1000_1	OK	982	2.934
knapPI_11_5000_1000_1	OK	2487	8.859		knapPI_13_5000_1000_1	OK	3057	9.204		knapPI_15_5000_1000_1	OK	2424	9.392
knapPI_12_20_1000_1	OK	8	0.027		knapPI_14_20_1000_1	OK	6	0.027		knapPI_16_20_1000_1	OK	6	0.027
knapPI_12_50_1000_1	OK	25	0.066		knapPI_14_50_1000_1	OK	25	0.065		knapPI_16_50_1000_1	OK	29	0.066
knapPI_12_100_1000_1	OK	43	0.132		knapPI_14_100_1000_1	OK	47	0.125		knapPI_16_100_1000_1	OK	44	0.126
knapPI_12_200_1000_1	OK	98	0.251		knapPI_14_200_1000_1	OK	3956	0.246		knapPI_16_200_1000_1	OK	103	0.254
knapPI_12_500_1000_1	OK	243	0.640		knapPI_14_500_1000_1	OK	283	0.643		knapPI_16_500_1000_1	OK	4049	0.638
knapPI_12_1000_1000_1	OK	480	1.337		knapPI_14_1000_1000_1	OK	524	1.340		knapPI_16_1000_1000_1	OK	485	1.326
knapPI_12_2000_1000_1	OK	1001	2.932		knapPI_14_2000_1000_1	OK	1061	2.903		knapPI_16_2000_1000_1	OK	979	2.941
knapPI_12_5000_1000_1	OK	2502	9.235		knapPI_14_5000_1000_1	OK	2526	9.259		knapPI_16_5000_1000_1	OK	5020	9.285

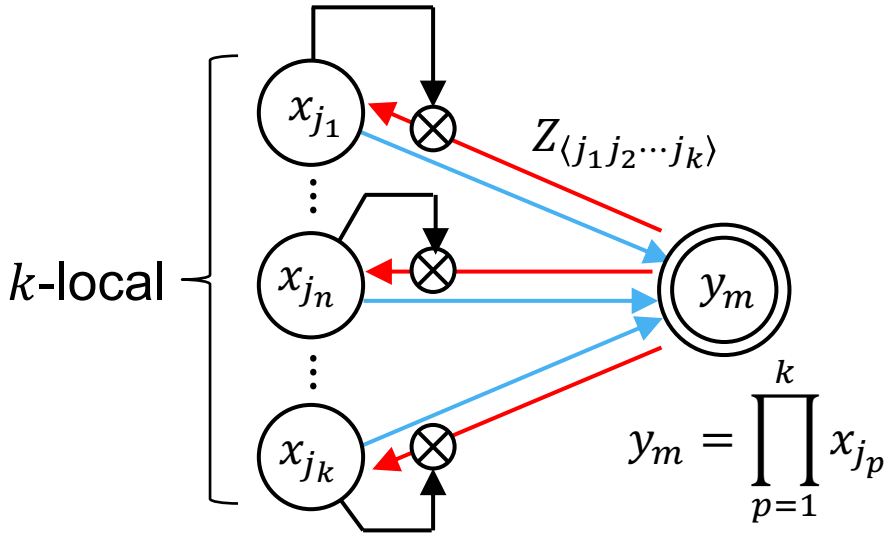
Application Example 2: Higher-Order Energy

- Parallel estimation of energy change is possible for higher order terms
  - Construct neural nets that generate local fields due to higher order terms
  - Auxiliary variables and local fields can be updated in parallel from local information

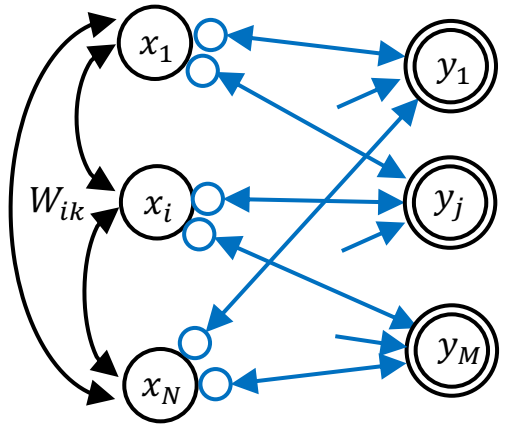
Local field due to k-body energy: $Z_{\langle j_1 j_2 \cdots j_k \rangle} \prod_{p=1}^{k} x_{j_p}$

$$Z_{\langle j_1 j_2 \cdots j_k \rangle} \left[x_{j_1} \cdots \left(x_{j_n} + \Delta x_{j_n} \right) \cdots x_{j_k} \right] - Z_{\langle j_1 j_2 \cdots j_k \rangle} \left[x_{j_1} \cdots x_{j_k} \right]$$

$$= \Delta x_{j_n} Z_{\langle j_1 j_2 \cdots j_k \rangle} \prod_{\substack{p=1 \\ p \neq n}}^{k} x_{j_p} = \underbrace{\Delta x_{j_n} Z_{\langle j_1 j_2 \cdots j_k \rangle} x_{j_n} \prod_{p=1}^{k} x_{j_p}}_{\text{Local field}}$$



Solving 3-SAT instances of SATLIB

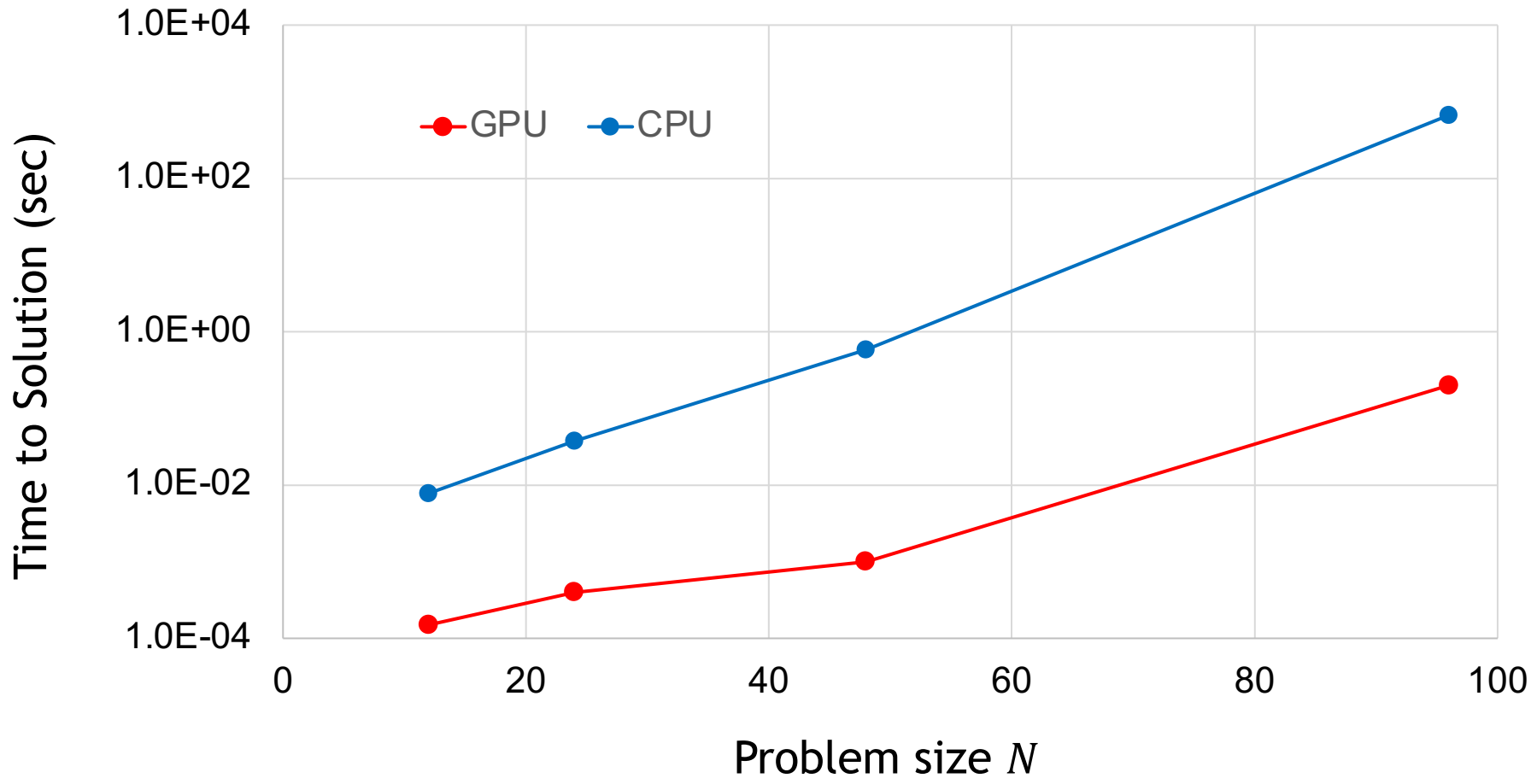


GPU (Titan V; 12GB)

Instance	N	M	Satisfied clauses	TTS (S)
Uf20-01	20	91	**91**	0.0012
Uf50-01	50	218	**218**	0.0019
Uf75-01	75	325	**325**	0.0025
Uf100-01	100	430	**430**	0.0033
Uf150-01	150	645	**645**	0.0053
Uf200-01	200	860	**860**	0.05
Uf250-01	250	1065	**1065**	0.08

3R3XORSAT Problems (3-body version)

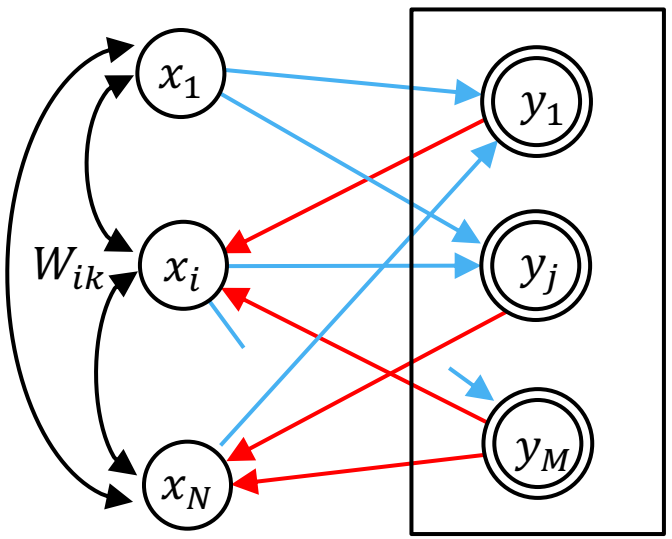
- Itay Hen's 3Regular 3XORSAT problems (3-local interaction)



Future Extension

- Embed auxiliary-variable neural nets within the Boltzmann Machine

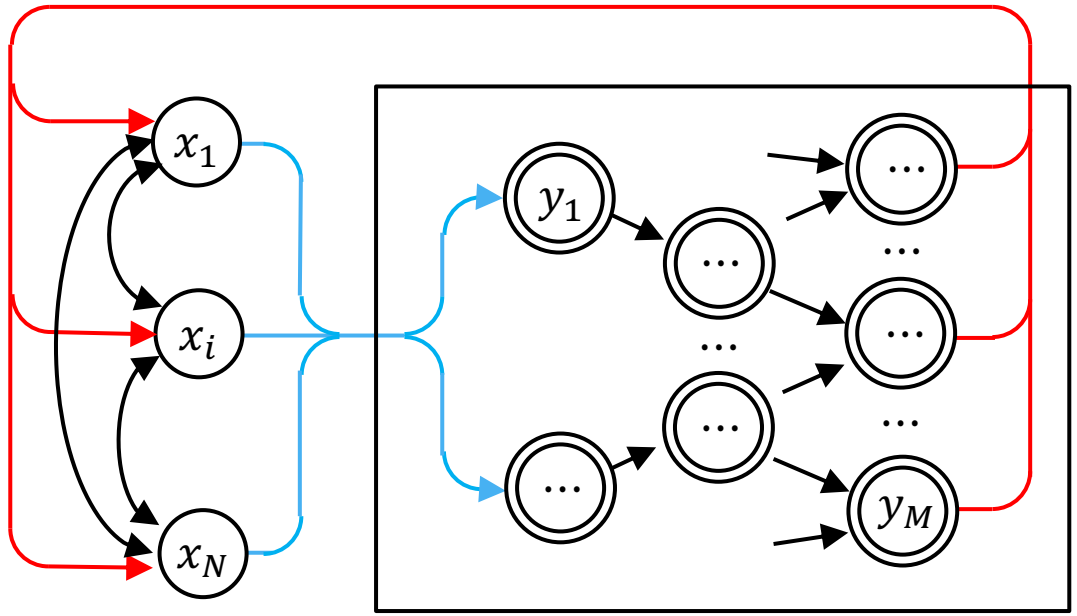
Single-stage auxiliary network



Auxiliary network

(a)

Multiple-stage auxiliary network

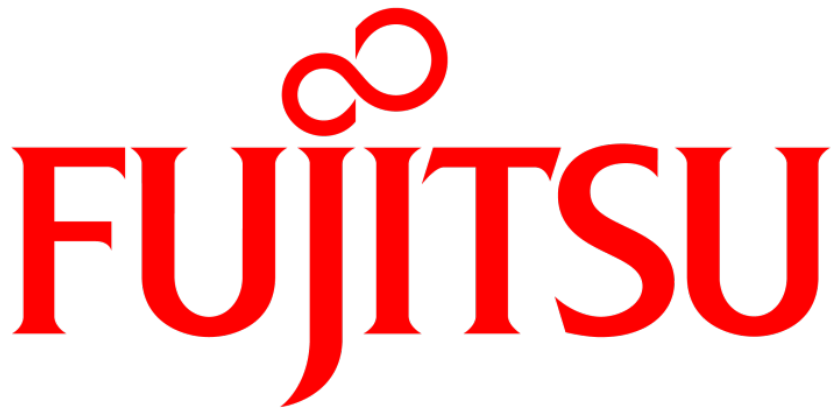


Auxiliary network

(b)

Conclusion

- We proposed a way to extend Ising machines by embedding an auxiliary neural network
- Proposed method extends the energy that the Ising machine can handle, which was previously limited to the binary quadratic form
  - Can handle multiple inequality constraints
  - Can handle k-local higher-order terms without increasing the number of decision variables
- These enhancements can retain the parallel trials with local fields and their parallel updates that have been the hallmarks of conventional DA
  - Speedup is possible with dedicated hardware



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