Energetic perspective on Rapid Quenches in Quantum Annealing

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Based on: **AC**[1,3], Max Festenstein[1,2], Jie Chen[2], Laurentiu Nita[2], Viv Kendon[2], Nicholas Chancellor[2], PRX Quantum, 2021

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- 'Problem Hamiltonian' $\hat{H}_{\mathrm{P}}$ diagonal in Pauli-$\hat{Z}$ basis, solution as ground-state
- Transverse field 'driver' $\hat{H}_{\mathrm{d}} = -\sum_{j=1}^{n} \hat{X}_j$
- Start in driver ground-state
- AQC-like parameterisation $\hat{H}_{AB}(t) = A(t)\hat{H}_{\mathrm{d}} + B(t)\hat{H}_{\mathrm{P}}$

$$A(0) = 1, B(0) = 0$$
$$A(t_f) = 0, B(t_f) = 1$$

- Change much faster than adiabatic timescales

- Equivalent parameterisation $\hat{H}_{\Gamma}(t) = \Gamma(t)\hat{H}_{\mathrm{d}} + \hat{H}_{\mathrm{P}}$

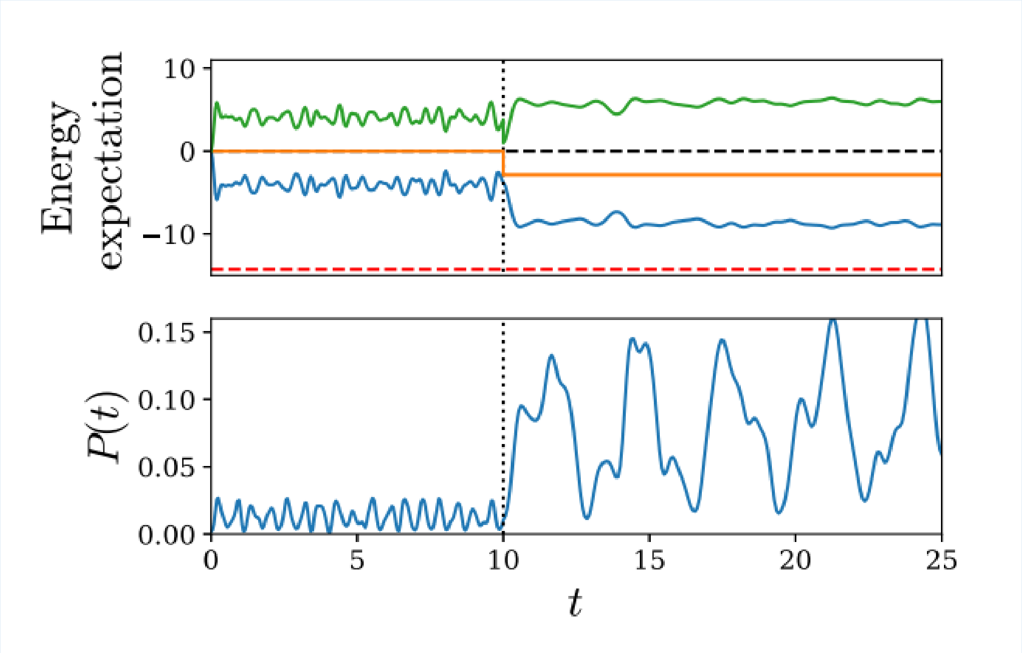
$$\Gamma(0) \gg 1, \Gamma(t_f) = 0$$

- Continuous-time quantum walks as starting point (see more in **AC**, N Chancellor, F Mintert, V Kendon, NJP, 2019)

$$\Gamma(t) = \gamma \left(= \begin{cases} \infty & t = 0 \\ \gamma & 0 < t < t_f \\ 0 & t = t_f \end{cases} \right)$$

- Two-stage quantum walk

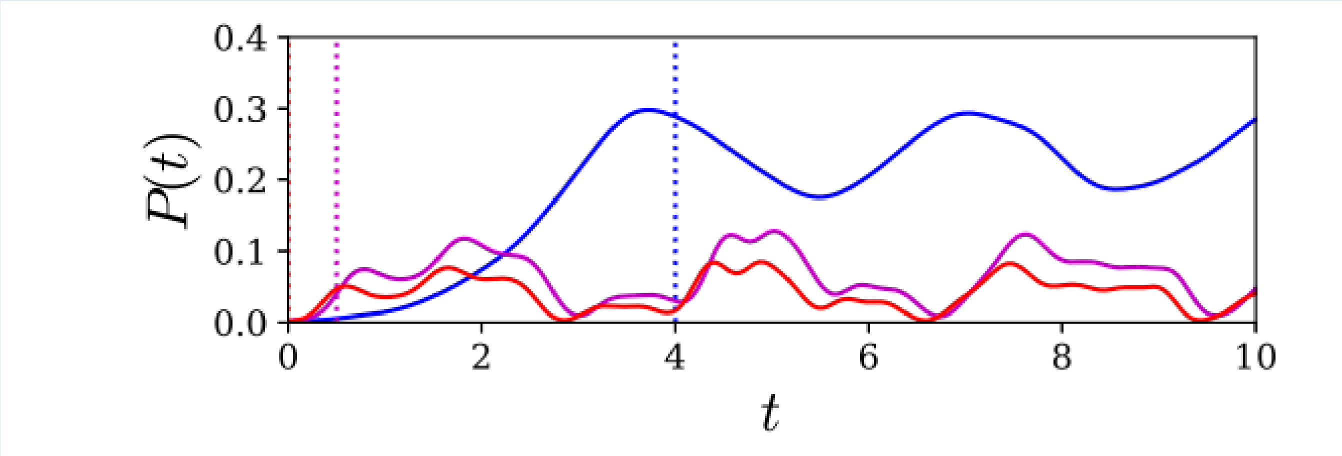
$$\Gamma(t) = \begin{cases} \gamma_1, & 0 < t < t_1 \\ \gamma_2, & t_1 \le t < (t_1 + t_2) \end{cases}$$



- Pre-annealed quantum walk

$$A(t) = \begin{cases} \gamma\left[1 + \left(\frac{t}{t_1} - 1\right)^2\right], & 0 < t < t_1 \\ \gamma, & t_1 \le t < (t_1 + t_2) \end{cases}$$

$$B(t) = \begin{cases} \left[1 + \left(\frac{t}{t_1} - 1\right)^2\right], & 0 < t < t_1 \\ 1, & t_1 \le t < (t_1 + t_2) \end{cases}$$



- In **AC**, N Chancellor, F Mintert, V Kendon, NJP, 2019 introduced very simple energy conservation mechanism to give intuition: energy expectation never made use by the quantum walk

$$\langle\psi(t)|\hat{H}|\psi(t)\rangle = \gamma\langle\psi(t)|\hat{H}_{\mathrm{d}}|\psi(t)\rangle + \langle\psi(t)|\hat{H}_P|\psi(t)\rangle$$

- Extend to time-dependent case with following extra conditions:
  - Control function positive $\Gamma(t) \geq 0$
  - Control function monotonic $\Gamma(t') \leq \Gamma(t)$ for $t' > t$
- Applies to many theoretical and experimental protocols (inc. AQC, QWs)
- Doesn't apply to non-monotonic 'reverse annealing'
- In practice, improvement is often large – so what else is going on?
- Examine fast dynamics

Dynamic Coefficient

- Inspect basis pairs directly connected by driver

$$\hat{H}_{\mathrm{P}}^{(j,k)} = \begin{pmatrix} \Delta_{jk} & 0 \\ 0 & 0 \end{pmatrix} \qquad \hat{H}_{\mathrm{d}}^{(j,k)} = \begin{pmatrix} \langle j|H_{\mathrm{d}}|j\rangle & \langle j|H_{\mathrm{d}}|k\rangle \\ \langle k|H_{\mathrm{d}}|j\rangle & \langle k|H_{\mathrm{d}}|k\rangle \end{pmatrix}$$

- Calculate 'Transfer Coefficient'

$$T^{(jk)} = R\left(\Gamma(t)\hat{H}_{\mathrm{d}}^{(j,k)}, \hat{H}_{\mathrm{P}}^{(j,k)}\right) = \frac{2\Gamma(t)|\langle k|H_{\mathrm{d}}|j\rangle|}{2\Gamma(t)|\langle k|H_{\mathrm{d}}|j\rangle| + |\Delta_{jk}|}$$

- Transform to basis of $\hat{H}_{\mathrm{d}}^{(j,k)}$ and calculate similar 'Driver coefficient'

$$D^{(jk)} = R\left(U^{(jk)\dagger}\hat{H}_{\mathrm{P}}^{(j,k)}U^{(jk)}, \Gamma(t)U^{(jk)\dagger}\hat{H}_{\mathrm{d}}^{(j,k)}U^{(jk)}\right)$$

- Take product $\chi^{(jk)} = T^{(jk)}D^{(jk)}$ and average over pairs $\bar{\chi} = \left\langle\chi^{(jk)}\right\rangle_{jk}$

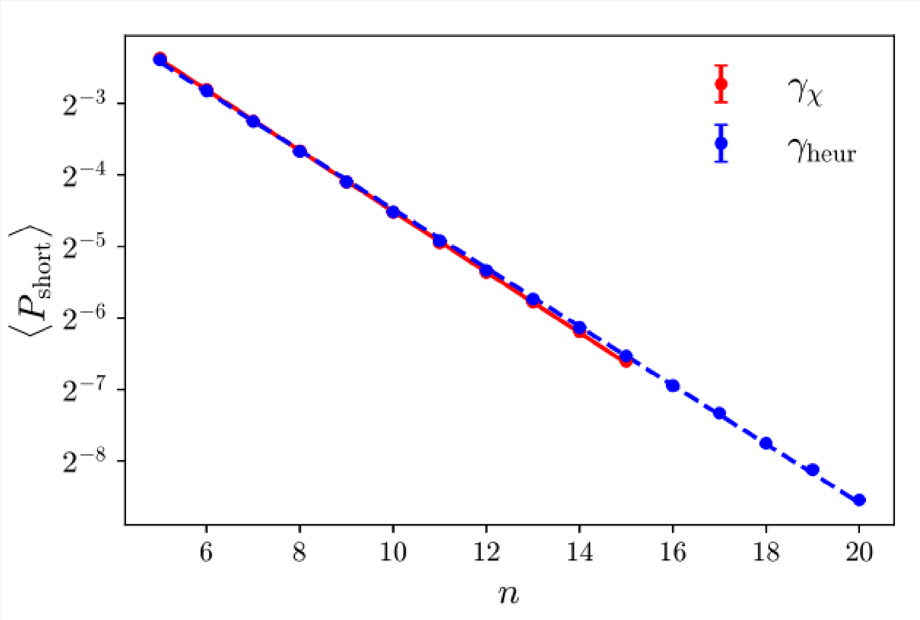
- Can use $\bar{\chi}$ to characterise whether a RQ could work

- Achieve small error with efficient number of sample pairs

- Some lower bounds based on Hamiltonian distribution for unbiased driver case

Heuristics for choosing quench parameters

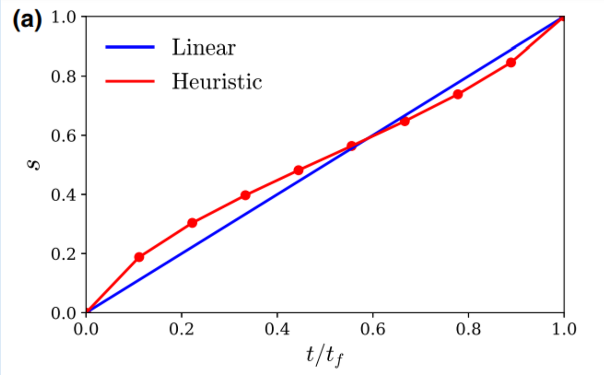
- Maximise $\bar{\chi}$ to find quantum walk hopping rate γ
- Tried on a spin-glass minimisation problem (see more in **AC**, N Chancellor, F Mintert, V Kendon, NJP, 2019)
- Performs almost as well as problem-specific heuristic from that paper

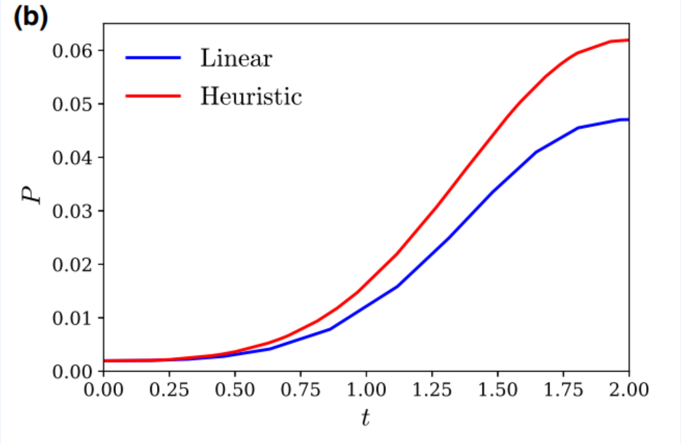


- Choose schedule shape for quantum annealing

$$A(t) = 1 - s(t), \qquad B(t) = s(t)$$

- Spend longer in high $\bar{\chi}$ regions





- Considered rapid quenches in quantum annealing, using continuous-time QWs as starting point
- Examples showing some improvements due to simple time-dependence
- Energy redistribution mechanism shows that time-dependent schedules will always improve in initial state
- Introduced dynamic coefficient as heuristic and often efficient way to characterize potential for a rapid quench to perform well
- Examples of using dynamic coefficient to heuristically choose parameters for rapid quenches