The Perturbed Ferromagnetic Chain: A Tuneable Test of Quantum Hardness in the Transverse-Field Ising Model

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Outline

1. Introduction to the Perturbed Ferromagnetic Chain
   - Motivation and Hamiltonian
   - Properties in the transverse-field Ising model
   - Semi-classical analysis

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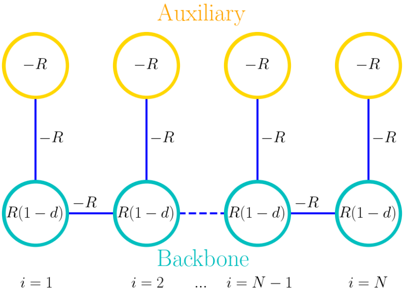
   - Motivation and Hamiltonian
   - Properties in the transverse-field Ising model
   - Semi-classical analysis
2. Simulation of the Perturbed Ferromagnetic Chain
   - Scaling with respect to system size
   - Comparing classical and quantum evolutions

Introduction to the Perturbed Ferromagnetic Chain

Perturbed Ferromagnetic Chain Hamiltonian



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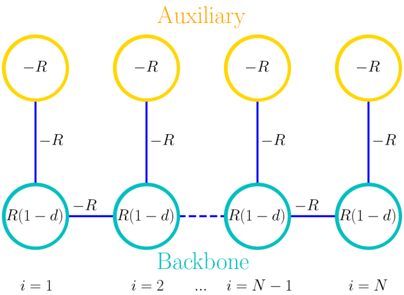
Perturbed Ferromagnetic Chain Hamiltonian



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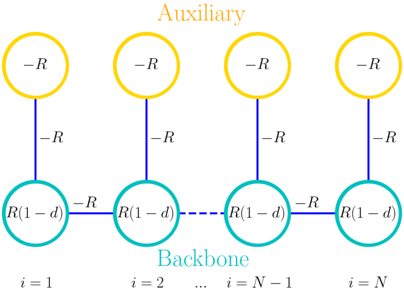
Perturbed Ferromagnetic Chain Hamiltonian



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  - Auxiliary qubits are iso-energetic to spin state

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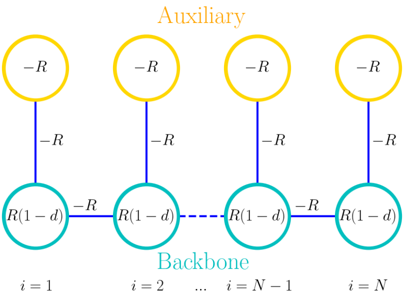

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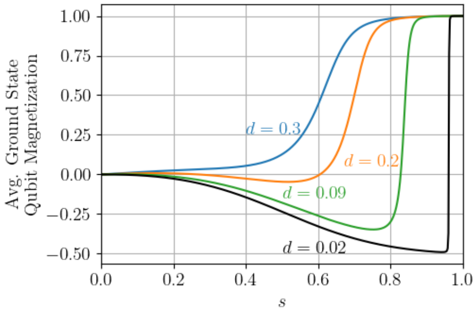
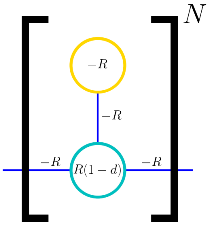
  - Auxiliary qubits are iso-energetic to spin state
- Always at least Hamming distance N from ground state

Transverse-Field Ising Model

Encode perturbed ferromagnetic chain into TFIM Hamiltonian

$$\hat{H}(s) = A(s)\sum_{j=1}^{2N}\hat{\sigma}_j^x + B(s)\hat{H}_{\mathrm{Problem}}\,,$$



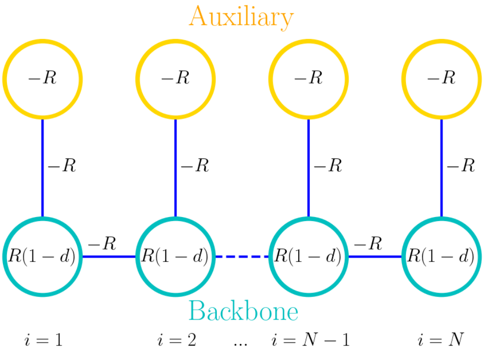


$N = 2,\ R = 1.0$

Schedules used are $A(s) = -3(1-s)$, and $B(s) = 3s$

Semi-Classical Analysis

- Approximate perturbed ferromagnetic chain with two angles for backbone (θ_b) and auxiliary (θ_a) qubits



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- The describe the perturbed ferromagnetic chain using the spin-coherent Ansatz in the XZ-plane of the Bloch sphere,

$$|\theta\rangle = \bigotimes_{j=1}^{2N} \cos\left(\frac{\theta_j}{2}\right)|0\rangle + \sin\left(\frac{\theta_j}{2}\right)|1\rangle$$

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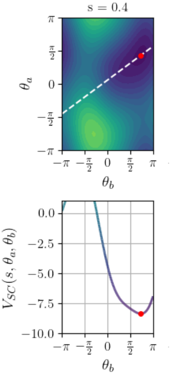
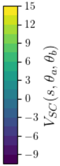
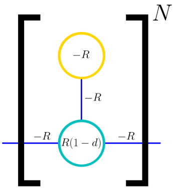
- Use this to visualise the semi-classical potential,

$$V_{SC}(s, \theta_a, \theta_b) = \langle\theta_a, \theta_b|\,\hat{H}(s)\,|\theta_a, \theta_b\rangle\,.$$

Semi-Classical Potential

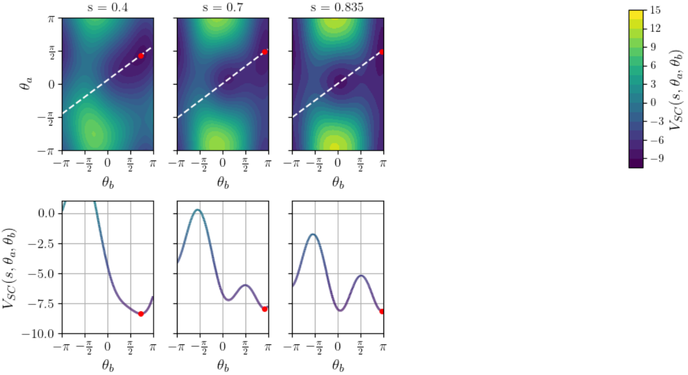


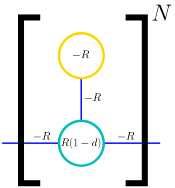




$N = 2,\ R = 1.0$
and $d = 0.09$

Semi-Classical Potential





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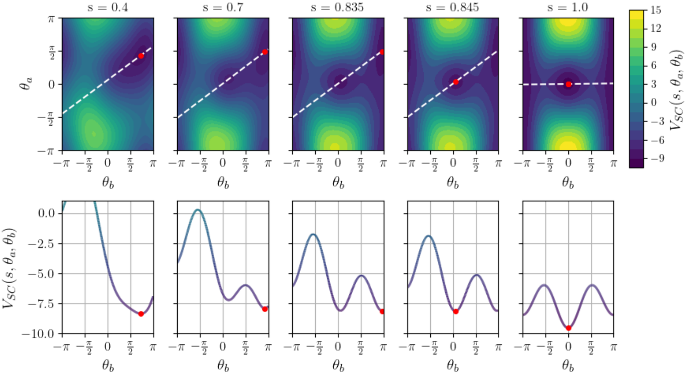
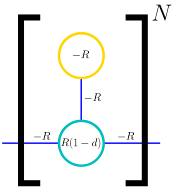
Minimum gap at $s = 0.841$

Semi-Classical Potential





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Dynamical Simulations of the Perturbed Ferromagnetic Chain

Simulation Methods

1. Spin-vector Monte Carlo (SVMC)[1]
2. Spherical-SVMC-TF - SVMC with transverse-field dependent updates[2] extended to include the azimuthal angle
3. Simulated quantum annealing (SQA)[3]
4. Adiabatic master equation (AME) using Hamiltonian open quantum system tool Kit (HOQST)[4]

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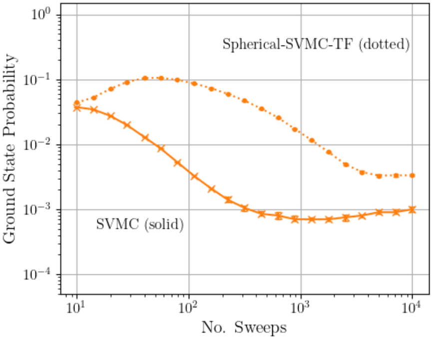
[1] Seung Woo Shin et al. “How "Quantum" Is the D-Wave Machine?” In: *arXiv:1401.7087 [quant-ph]* (May 2014). arXiv: 1401.7087 [quant-ph].

[2] Tameem Albash and Jeffrey Marshall. “Comparing Relaxation Mechanisms in Quantum and Classical Transverse-Field Annealing”. In: *Physical Review Applied* 15.1 (Jan. 2021), p. 014029. ISSN: 2331-7019. DOI: 10.1103/PhysRevApplied.15.014029. arXiv: 2009.04934.

[3] Roman Martoňák, Giuseppe E. Santoro, and Erio Tosatti. “Quantum Annealing by the Path-Integral Monte Carlo Method: The Two-Dimensional Random Ising Model”. In: *Physical Review B* 66.9 (Sept. 2002), p. 094203. DOI: 10.1103/PhysRevB.66.094203.

[4] Huo Chen and Daniel A. Lidar. “HOQST: Hamiltonian Open Quantum System Toolkit”. In: *arXiv:2011.14046 [quant-ph]* (Nov. 2020). arXiv: 2011.14046 [quant-ph].

Spin-Vector Monte Carlo Dynamics



Annealing schedules are $A(s) = -3(1 - s)$ and $B(s) = 3s$ at a temperature of 12mK

- Ground state probability after a n sweep anneal for a perturbed ferromagnetic chain with $d = 0.1$ and $N = 3$

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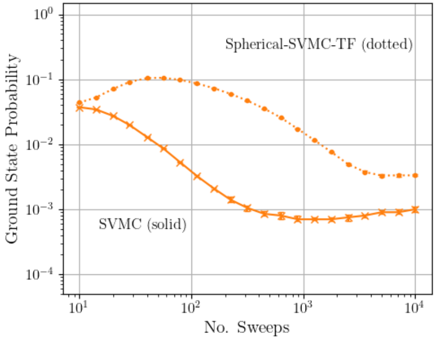

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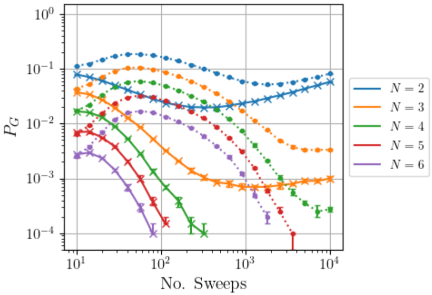
- Ground state probability after a n sweep anneal for a perturbed ferromagnetic chain with $d = 0.1$ and $N = 3$
- Anneal experiences 3 regimes as anneal sweeps increase
  1. Discrete evolution guides spin-vector to low-energy states
  2. Quasi-continuous evolution takes spin-vector down false minimum
  3. Thermal equilibration

Scaling with N



SVMC (solid line) and
Spherical-SVMC-TF (dotted line)

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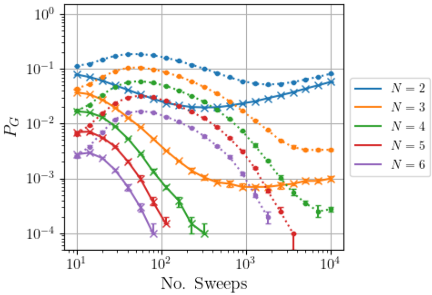

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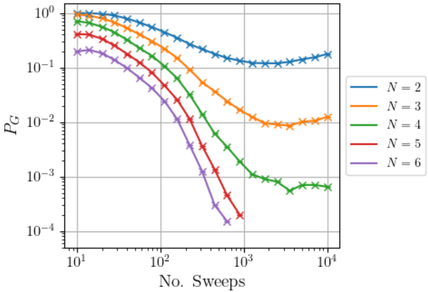
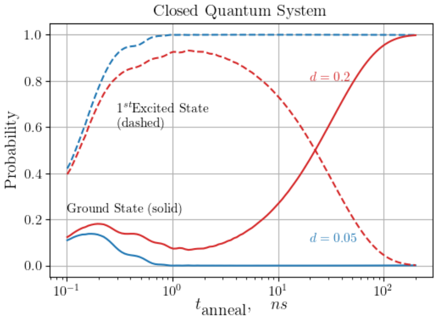
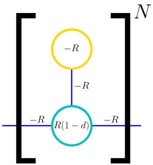


Simulated quantum annealing with 128 discrete slices annealed under the same parameters as SVMC

Quantum Evolution





$N = 3,$
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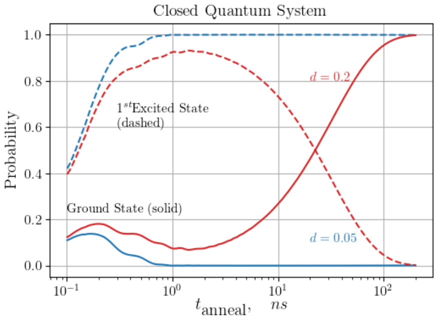
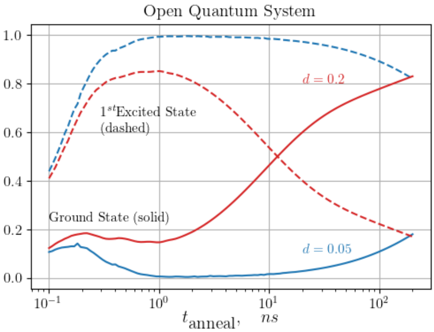
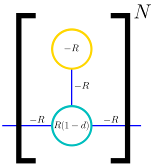
Quantum Evolution







$N = 3$,
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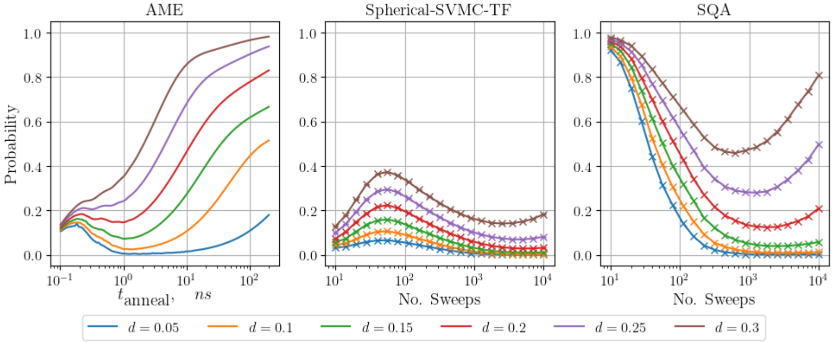
- Open system simulations implement decoherence by dephasing.

Ohmic bath with $T = 12$ mK, $\omega_c = 4$ GHz and $\eta g^2 = 10^{-3}$

Comparing Evolutions, $N = 3$



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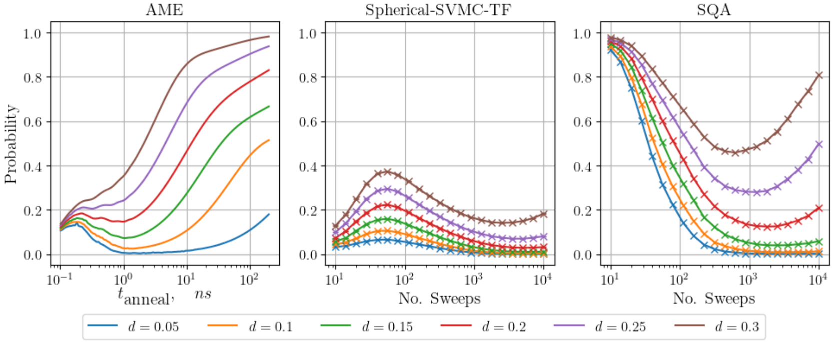

$d = 0.05$	AME	Spherical-SVMC-TF	SQA	Thermal Eq.
Anneal Time	100 ns	10,000 Sweeps	10,000 Sweeps	-
Probability	~ 0.1	$\sim 10^{-3}$	$\sim 2 \times 10^{-3}$	0.82

Conclusions

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- Follows a false minimum in the transverse-field Ising model

- Classical evolution struggles with the false minimum leading to the first excited state manifold
- Quantum evolution remains in the lowest two energy eigenstates
- Scope for scaling experiments on NISQ hardware

Questions

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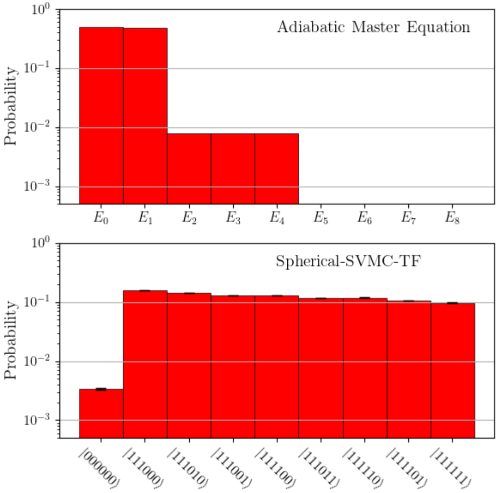
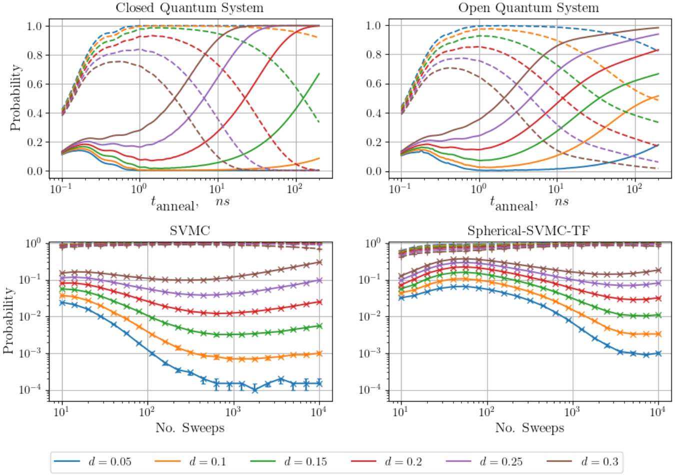
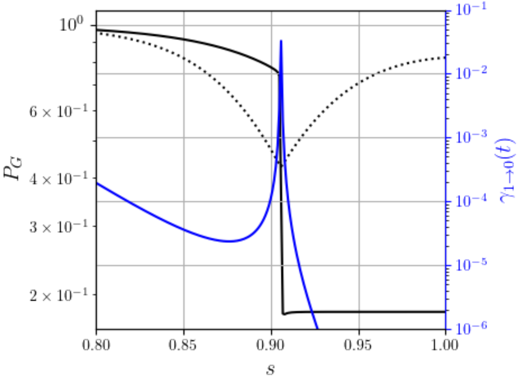
State Distributions



Hardness with d



Transition Rate



$$\gamma_{1\to0}(t) = \gamma(\omega_{10}(t)) \sum_i |\langle E_0(t)|\sigma_i^z|E_1(t)\rangle|^2 \ .$$